Radiation

Jintai Lin

北京大学
PEKING UNIVERSITY

Outline

- Introduction
- Concepts
- Absorption
- Scattering
- Radiative transfer
- Radiative equilibrium temperature
- Radiative heating and cooling

致谢：本课件中许多资料来自李成才老师
（特别是关于辐射的部分）。

思考题

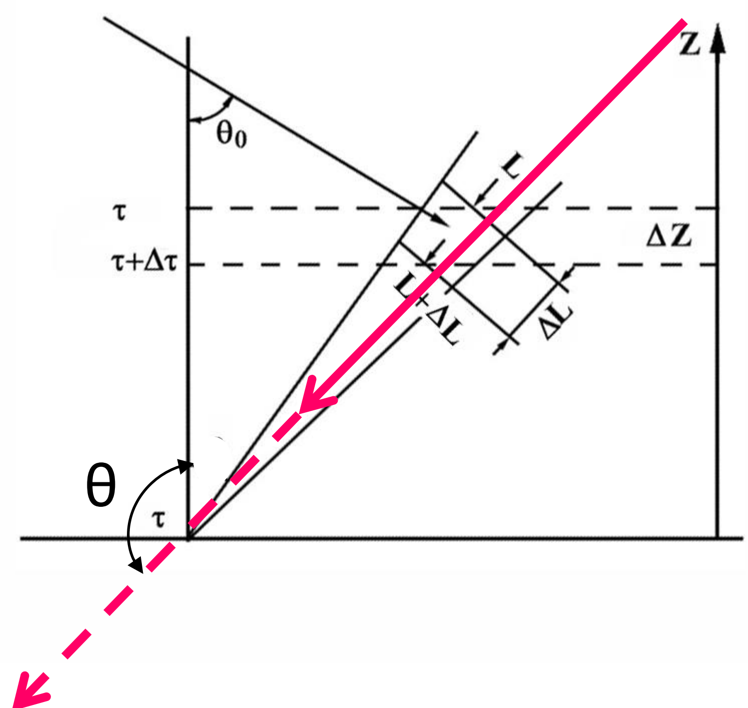
- 问题一：说明不对称因子g的意义：g > 0, g < 0, g = 0。假设散射时向上和向下的散射光强分别是一样的（semi-isotropic），在这种情况下，g可能达到的最大值是多少？g可能达到1么？g= 1和g=-1分别意味着什么？

- 问题二：‘蓝月亮’是指月亮呈现蓝色，是很罕见的现象。思考肉眼从地面向上看到‘蓝月亮’所需条件，以及为什么这种现象很少出现。

https://zhuanlan.zhihu.com/p/33472940
https://m.sohu.com/n/464038572/?_f=m-article_ch

Radiative Transfer 辐射传输

- 当我们观察天空某一方向的亮度大小，实际上是接收在这个立体角中自眼睛直到大气上界整个气柱所发来的光。
- 让我们分析经过一段距离 Δl，气柱辐亮度的变化 ΔL。



辐射传输

- 考察某一波长的辐亮度在经过一段气柱后的变化，这种变化是由下列4种因素引起的：

① *L*经过这段气柱后受到衰减；

②由于太阳光直接射到这段气柱上，气柱发出散射光射向光度计，即一次散射

③气柱周围各个方向的散射光射到这段气柱上再发生散射，即多次散射；

④ 这段气柱中大气的热辐射。

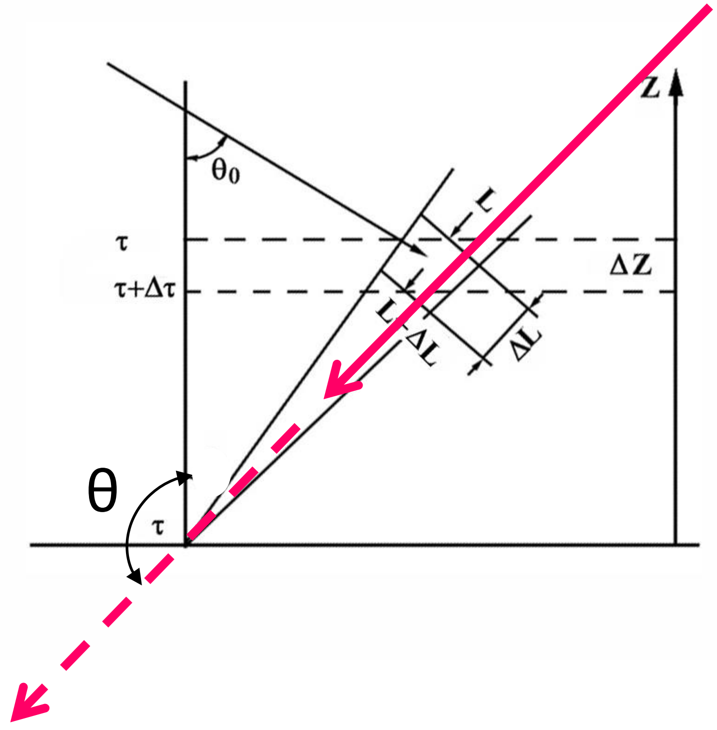
Bill定律：e指数衰减

【Bill定律只考虑吸收和一次散射的作用】

$$dI = -I \cdot n\sigma_{ex}\, dl$$

$$dI = -I \cdot k_{ex} \cdot dl$$

$$k_{ex} = k_{ab} + k_{sc} = n\sigma_{ab} + n\sigma_{sc}$$



这里省略波长标识

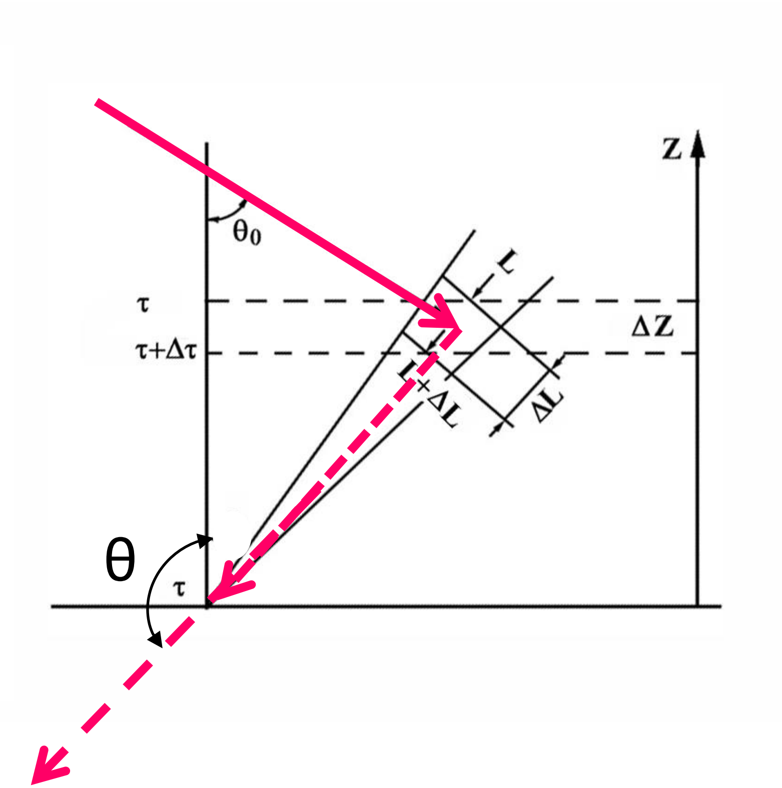
一次散射

$$dF_s(\theta,\phi) = \frac{F_{\odot}}{R^2}\beta(\theta,\phi)\,dv$$

$$dI_s \cdot \Delta\Omega = \frac{F_{\odot}}{R^2}\beta(\theta,\phi)\cdot \Delta S \cdot dl$$

$$dI_s \cdot \Delta\Omega = F_{\odot}\cdot \beta(\theta,\phi)\cdot dl \cdot \Delta\Omega$$

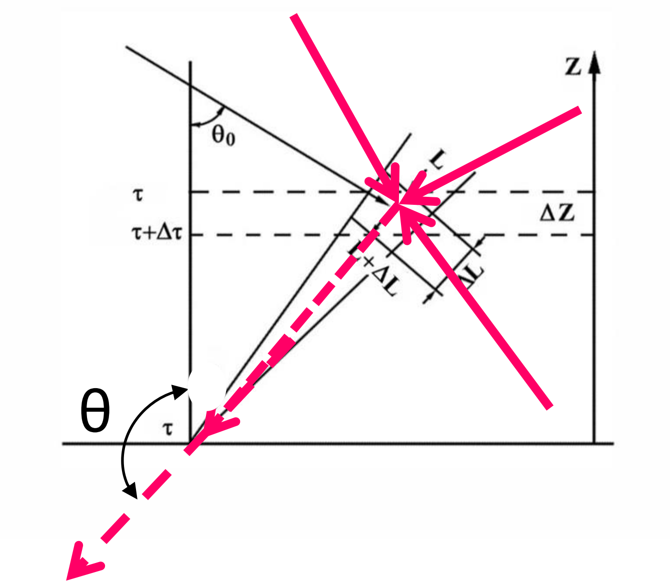
$$dI_s = F_{\odot}\cdot \beta(\theta,\phi)\cdot dl$$



对于太阳辐射的一次散射： $F_{\odot} = F_0\, e^{-\tau(z)/\cos(\theta_0)}$

这里省略波长标识

多次散射



- **单次散射**

$$dI_s = F_{\odot} \cdot \beta(\theta,\phi) \cdot dl$$

- **多次散射：**
  入射光为四面八方来的散射光：

$$dF_{\odot} = I \cdot d\Omega'$$

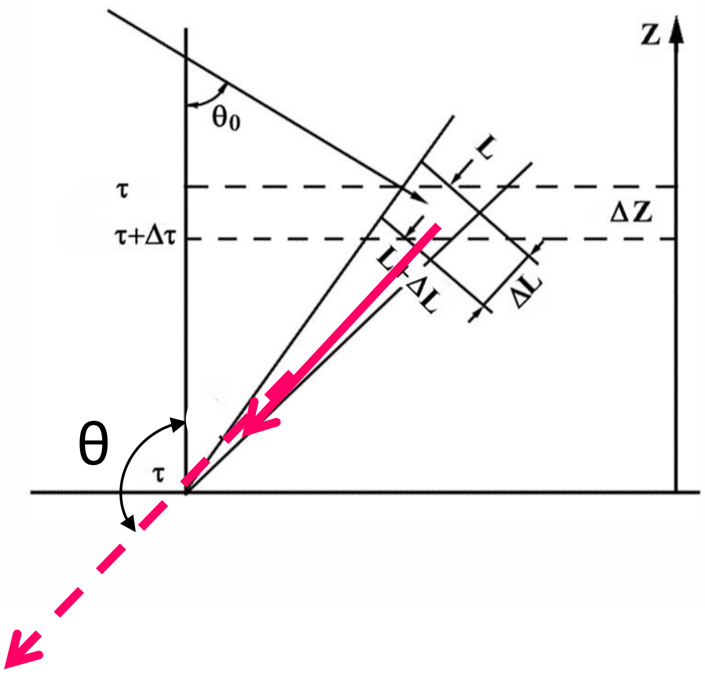
$$dI_s = \int_0^{2\pi}\int_0^{\pi} I(z,\theta',\varphi')\beta(z,\theta,\varphi,\theta',\varphi')\sin\theta' \mathrm{d}\theta' \mathrm{d}\varphi' \cdot dl$$

这里省略波长标识，假设平面平行大气

$\odot$ **表示照射到小气块的辐射**

热辐射

- **放射：**
  即热辐射，
  由普朗克定律
  和基尔霍夫定律确定：



$$dI = B(T) \cdot k_{ab}(z) \cdot dl$$

这里省略波长标识

大气中的辐射传输方程

$$dI = -I(z,\theta,\varphi)k_{ex}dl$$

$$+ F_0 \, \mathrm{e}^{-\tau(z)/\cos(\theta_0)} \, \beta(z,\theta,\varphi,\theta_0,\varphi_0)\cdot dl$$

$$+\int_0^{2\pi}\int_0^{\pi} I(z,\theta',\varphi')\beta(z,\theta,\varphi,\theta',\varphi')\sin\theta'\mathrm{d}\theta'\mathrm{d}\varphi' dl$$

$$+ B[T(z)]k_{ab}(z)dl$$

这里省略波长标识，假设平面平行大气

F_0表示太阳短波辐射

大气中的辐射传输方程

平面平行大气：$dl = dz / \cos\theta$；水平方向均一

用：$\mu = \cos\theta$， $\mu_0 = \cos\theta_0$

相函数 $P(\Theta) = 4\pi\beta(\Theta)/k_{sc}$,

单散射反照率 $\omega_0 = k_{sc}/k_{ex}$

并用光学厚度坐标代替z坐标，有

$$d\tau = -k_{ex}\, dz$$

τ 坐标从大气上界算起，向下逐渐增加，到地面为 τ_0，即整层大气垂直光学厚度

这里省略波长标识

Note the definition of θ_0

大气中的辐射传输方程

Schwarzschild 史瓦西传输方程（变形形式），J 称为源函数

$$\mu \frac{\mathrm{d} I}{\mathrm{d} \tau} = I - J$$

$$J = \frac{\omega_0}{4\pi} \left\{ F_0 \cdot \mathrm{e}^{-\tau / \mu_0} P(\Theta_0) + \int_0^{2\pi} \int_{-1}^{1} I(\tau, \mu', \varphi') P(\tau, \mu, \varphi, \mu', \varphi') \mathrm{d}\mu' \mathrm{d}\varphi' \right\} + (1-\omega_0) B[T(\tau)]$$

大气中的辐射传输方程

- 其中Θ_0是从太阳入射光方向（$\pi-\theta_0$，$\varphi_0-\pi$）到观测到辐射的来向（θ，φ）之间的夹角，即散射角

$$\cos\Theta_0 = -\cos\theta\cos\theta_0 - \sin\theta\sin\theta_0\cos(\varphi-\varphi_0)$$

$$= -\mu\mu_0 - \sqrt{1-\mu^2}\sqrt{1-\mu_0^2}\cos(\varphi-\varphi_0)$$

Note the definition of θ_0 and φ_0

太阳高度角　　太阳方位角

大气中的辐射传输方程

- 上式即为**平面平行大气中的辐射传输方程**。其中J称为源函数，包括一次散射，多次散射和热辐射项。
- 在不同的问题中J_λ可以作相应简化：
  - 对短波辐射，热辐射项可以不考虑
  - 对晴空、清洁条件下的红外辐射，散射项可以不考虑
  - 讨论红外辐射在云中传输这类问题时，则一次散射项有时可以不计

大气中的辐射传输方程

- 上面推导辐射传输方程时一个最主要的简化是假定也称平面平行大气。
- 在很多情况下这一条件可以认为是能满足的，因此平面平行大气辐射传输方程在讨论许多问题时被广泛的应用。
- 但在处理有些大气物理的问题，例如天空有不均匀分布的云或讨论曙暮光这类必须考虑球面大气的问题时，就必须应用三维空间的辐射传输方程了。

Solution to Radiative Transfer: Thermal Radiation

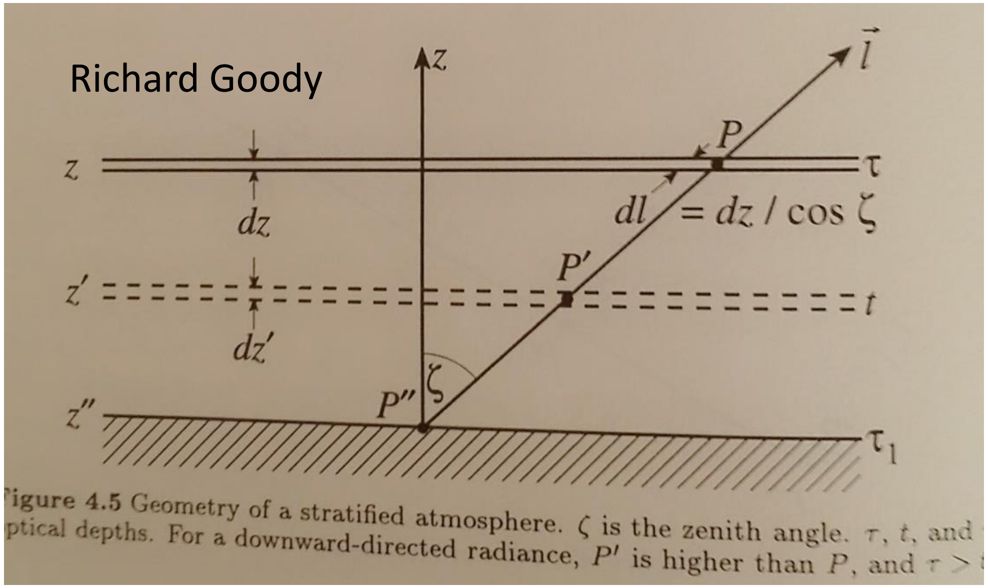
Integral equation for thermal radiation:

- **No solar radiation**
- **No scattering of thermal radiation**

$$J = B[T(\tau)]$$

$$\mu \frac{\mathrm{d}I}{\mathrm{d}\tau} = I - J$$

$$= I - B[T(\tau)]$$



We get:

$$I = I_i \cdot e^{-(\tau_i - \tau)/\mu} + 1/\mu \cdot \int_{\tau}^{\tau_i} B[T(\tau')] \cdot e^{-(\tau' - \tau)/\mu} d\tau'$$

Upward and Downward Radiance

We distinguish upward (I^+) and downward (I^-) radiance

Upper and lower boundary conditions:

$$I^-(\tau = 0) = 0 \qquad \text{for } -1 \le \mu \le 0$$

$$I^+(\tau = \tau_1) = B[T_g] \qquad \text{for } 0 \le \mu \le +1 \text{ ; g is ground}$$

We therefore obtain:

$$I^+ = B[T_g] \cdot e^{-(\tau_1 - \tau)/\mu} + 1/\mu \cdot \int_{\tau}^{\tau_1} B[T(\tau')] \cdot e^{-(\tau' - \tau)/\mu} d\tau'$$

$$I^- = -1/\mu \cdot \int_0^{\tau} B[T(\tau')] \cdot e^{-(\tau' - \tau)/\mu} d\tau'$$

- **I^+ and I^- are always positive values**

Flux Density

$$F = \int_{4\pi} I \cdot \mu \cdot d\Omega$$

$$= \int_{-1}^{+1} \int_{2\pi} I \cdot \mu \cdot d\phi \cdot d\mu$$

Stratified

$$= \int_{-1}^{+1} 2\pi \cdot I \cdot \mu \cdot d\mu$$

μ = cos(θ), where θ = zenith angle

Upward and Downward Flux Density

$$F^{+} = \int_{0}^{1} 2\pi \cdot I^{+} \cdot \mu \cdot d\mu$$

$$F^{-} = \int_{0}^{-1} 2\pi \cdot I^{-} \cdot \mu \cdot d\mu$$

- **F: (Net) flux density. Positive or negative value**
- **F^{+}: Upward flux density. Positive value**
- **F^{-}: Downward flux density. Positive value**

Flux Density

$$F = \int_{-1}^{+1} 2\pi \cdot I \cdot \mu \cdot d\mu$$

F⁺

$$= 2\pi \cdot B[T_g] \cdot E_3(\tau_1 - \tau)$$

$$+ 2\pi \cdot \int_{E_3(\tau_1-\tau)}^{1/2} B[T(\tau')]dE_3(\tau'-\tau)$$

F⁻

$$- 2\pi \cdot \int_{E_3(\tau)}^{1/2} B[T(\tau')]dE_3(\tau'-\tau)$$

Third exponential integral:

$$E_3 = \int_0^1 y \cdot e^{-x/y} dy$$

Radiance to Space Approximation: *Radiative Heating*

For thermal radiation only, with no solar radiation & scattering

- **Upper edge layer: loosing energy to space**
- **Middle layers: inward and outward radiation cancels to some degree, with certain portion towards outer space**
- **Air temperature in lower and mid atmospheres are normally within 40 K of the mean**

To calculating radiative heating in a layer of interest, assuming all layers and the ground have the same temperature as that level, so that all radiation exchange, except with space, is identically zero. Therefore:

$$I^{+} = B[T(\tau)] \qquad \text{for } 0 \le \mu \le +1$$

$$I^{-} = B[T(\tau)] \cdot \left[1 - e^{\tau/\mu}\right] \qquad \text{for } -1 \le \mu \le 0$$

Radiance to Space Approximation

Thermal radiation heating rate (per unit volume):

$$\rho \frac{dq}{dt} = -\frac{dF}{dz} = -\int_{-1}^{+1} 2\pi \cdot \frac{dI}{dz} \cdot \mu \cdot d\mu$$

$$= -\int_{-1}^{+1} 2\pi \cdot \frac{dI}{dl} \cdot d\mu$$

$$= \int_{-1}^{+1} 2\pi \;\; k_{ab} \left(I - B \right) \cdot d\mu$$

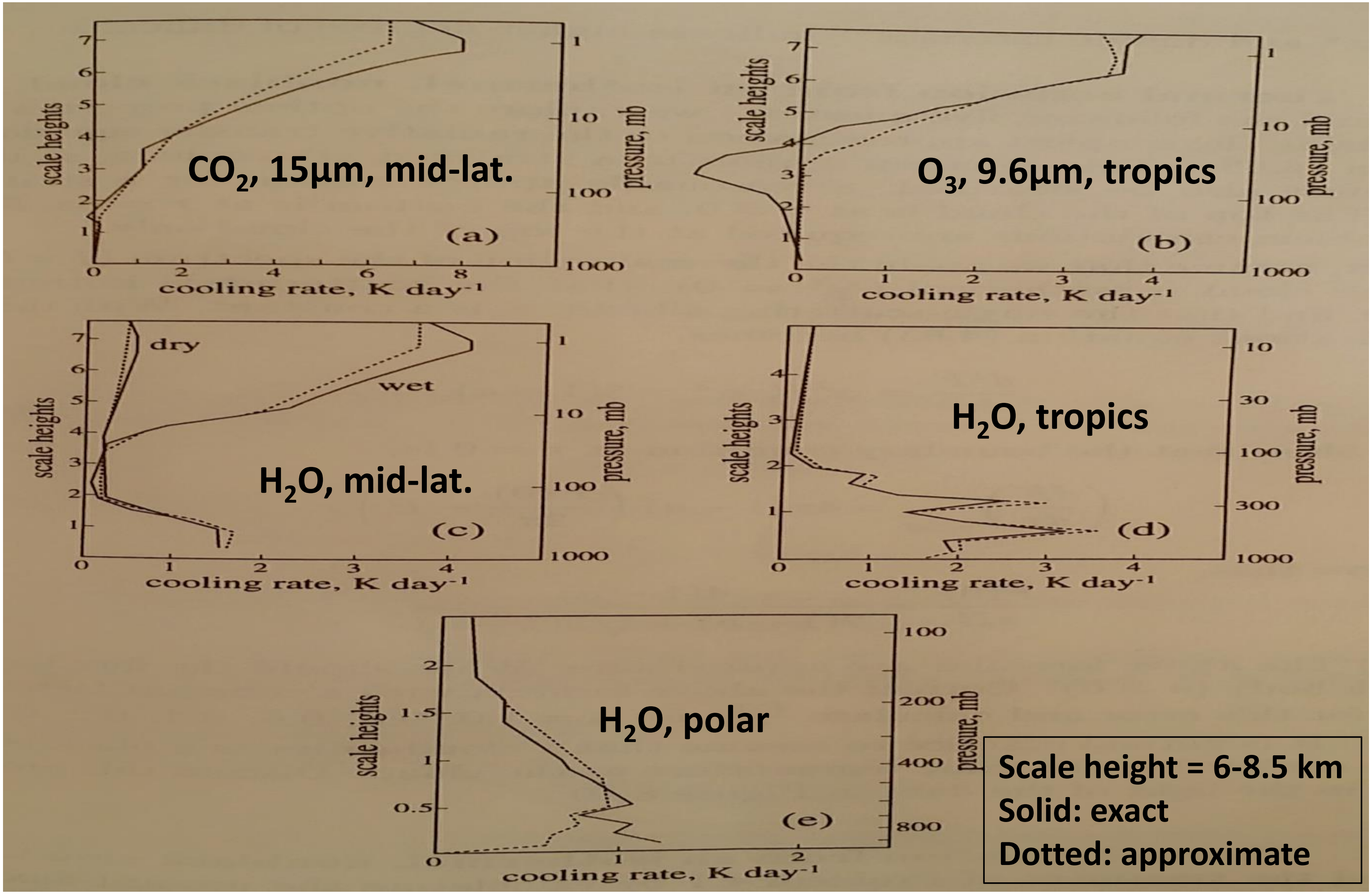
$$= \int_{0}^{+1} 2\pi \;\; k_{ab} \left(I^{+} - B \right) \cdot d\mu + \int_{-1}^{0} 2\pi \;\; k_{ab} \left(I^{-} - B \right) \cdot d\mu$$

$$= -\int_{-1}^{0} 2\pi \;\; k_{ab} \, B \cdot e^{\tau / \mu} d\mu$$

$$= -2\pi \cdot k_{ab} \, B(\tau) \cdot E_2(\tau)$$

Second exponential integral: $E_2(x) = \int_0^1 e^{-x/y} dy$

Radiance to Space Heating Rate



Scale height = 6-8.5 km
Solid: exact
Dotted: approximate

Richard Goody

Approximate Differential Equations for Diffuse Flux

- **For many problems one concerns radiation flux in a stratified atmosphere only**
- **Direct solar radiance can be easily calculated with Bill's Law, thus not needed to be included in the radiative transfer equation**

$$\mu \frac{dI}{d\tau} = I - J$$ **(Diffuse radiance, stratified atmosphere)**

$$J = (1-a)B + {a}/{4\pi} \int_{4\pi} I(\mu', \phi')P(\mu, \phi; \mu', \phi')d\Omega' + {a}/{4\pi} \cdot f \cdot P(\mu, \phi, \mu_0, \phi_0)$$

$$f = F_0 \cdot e^{-\tau/\mu_0}$$

- **a = SSA**
- **$\mu_0 = \cos(\theta_0) > 0$**
- **Suffix '0' indicates solar irradiance**

Method of Moments, Semi-Isotropic Field of Radiance

矩量法

Method of moments:

定义 $$\int_{4\pi} I d\Omega = 4\pi \bar{I}$$

定义 $$\int_{4\pi} I\mu d\Omega = F$$

近似 $$\int_{4\pi} I\mu^2 d\Omega \approx {4\pi}/{3} \cdot \bar{I}$$

Semi-isotropic Radiance:

$$\bar{I} = (I^+ + I^-)/2$$

$$F = \pi(I^+ - I^-)$$ **Upward: positive**

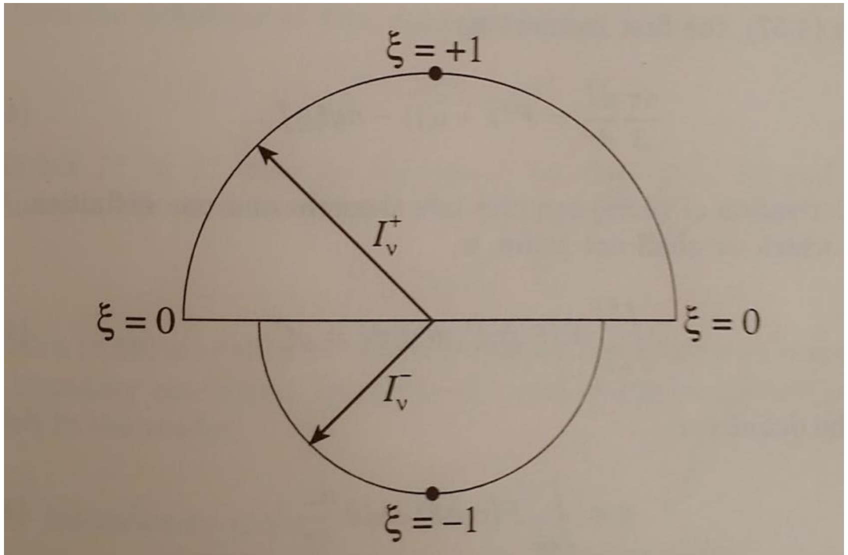


Figure 4.10 A semi-isotropic field of radiation.

Method of Moments, Semi-Isotropic Field of Radiance

We get:

$$\frac{dF}{d\tau} = 4\pi(1-a)(\bar{I} - B) - af$$

$$4\pi/3 \cdot \frac{d\bar{I}}{d\tau} = F(1-ag) - agf\mu_0$$

Here:

$$g = 1/4\pi \int_{4\pi} P(\mu)\mu d\Omega$$

$$\int_{-1}^{+1} P(\mu, \phi; \mu', \phi')\mu d\mu = 2g\mu'$$

Thus:

Diffuse radiation

$$\frac{d^2F}{d\tau^2} = 3(1-a)(1-ag)F - 4\pi(1-a)\frac{dB}{d\tau} - f\left[a/\mu_0 + 3ag\mu_0(1-a)\right]$$

Radiative Transfer of Diffuse Flux

$$\frac{d^2F}{d\tau^2} = 3(1-a)(1-ag)F - 4\pi(1-a)\frac{dB}{d\tau} - f\left[{a}/{\mu_0} + 3ag\mu_0(1-a)\right]$$

Lower boundary conditions:

$$I^+ = B_g$$

$$F = 2\pi(B_g - \bar{I})$$

Upper boundary conditions:

$$I^- = 0$$

$$F = 2\pi\bar{I}$$

思考题

4.1 **Thermal emission from an isothermal, nonblack cloud, I.** In this and the following five questions we explore the optical properties of clouds using the simplest approximation to the radiative transfer equation, equation (4.63). This equation applies to a stratified atmosphere, so the cloud must also be stratified, a reasonable approximation for a stratus cloud. The top of the cloud is at $\tau = 0$, and the bottom is at $\tau = \infty$. For this problem calculations are required at the top of the cloud only.

First, assume that we are in the thermal region of the spectrum ($f = 0$), that the cloud is isothermal ($\frac{dB}{d\tau} = 0$), that the scattering is isotropic, ($g = 0$), and that the single-scattering albedo, a, is a constant. With these simplifications equation (4.63) becomes,

$$\frac{d^2F}{d\tau^2} = \alpha^2 F, \ \alpha^2 = 3(1-a).$$

(i) Show that the boundary condition at $\tau = 0$ is,

$$\left(\frac{dF}{d\tau}\right)_{\tau=0} = 4\pi(1-a)\left(\frac{F(0)}{2\pi} - B\right).$$

Show that,

$$\frac{F(0)}{\pi B} = \frac{4(1-a)}{2(1-a)+\sqrt{3(1-a)}}.$$

(ii) The above formulae are approximate. What should the flux be for a black body ($a = 0$)? Correct the above formula with a constant factor to allow for this error and calculate $\frac{F(0)}{\pi B}$ for $a = 1.0,\ 0.8,\ 0.6,\ 0.4,\ 0.2,\ 0.0$.

(iii) It is normal practice to assume that a cloud emits as a black body whose surface has the same temperature as the cloud. Discuss this proposition in the light of the data in Figure 8.10.

4.2 **Thermal emission from an isothermal, nonblack cloud, II.** Extend the treatment of Problem 4.1 by calculating the upward flux for the same values of a, but with $g = 0.5, 1.0$. Comment upon the singular case $g = 1$.