Radiation

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致谢：本课件中部分资料来自李成才老师（特别是关于辐射的部分）。

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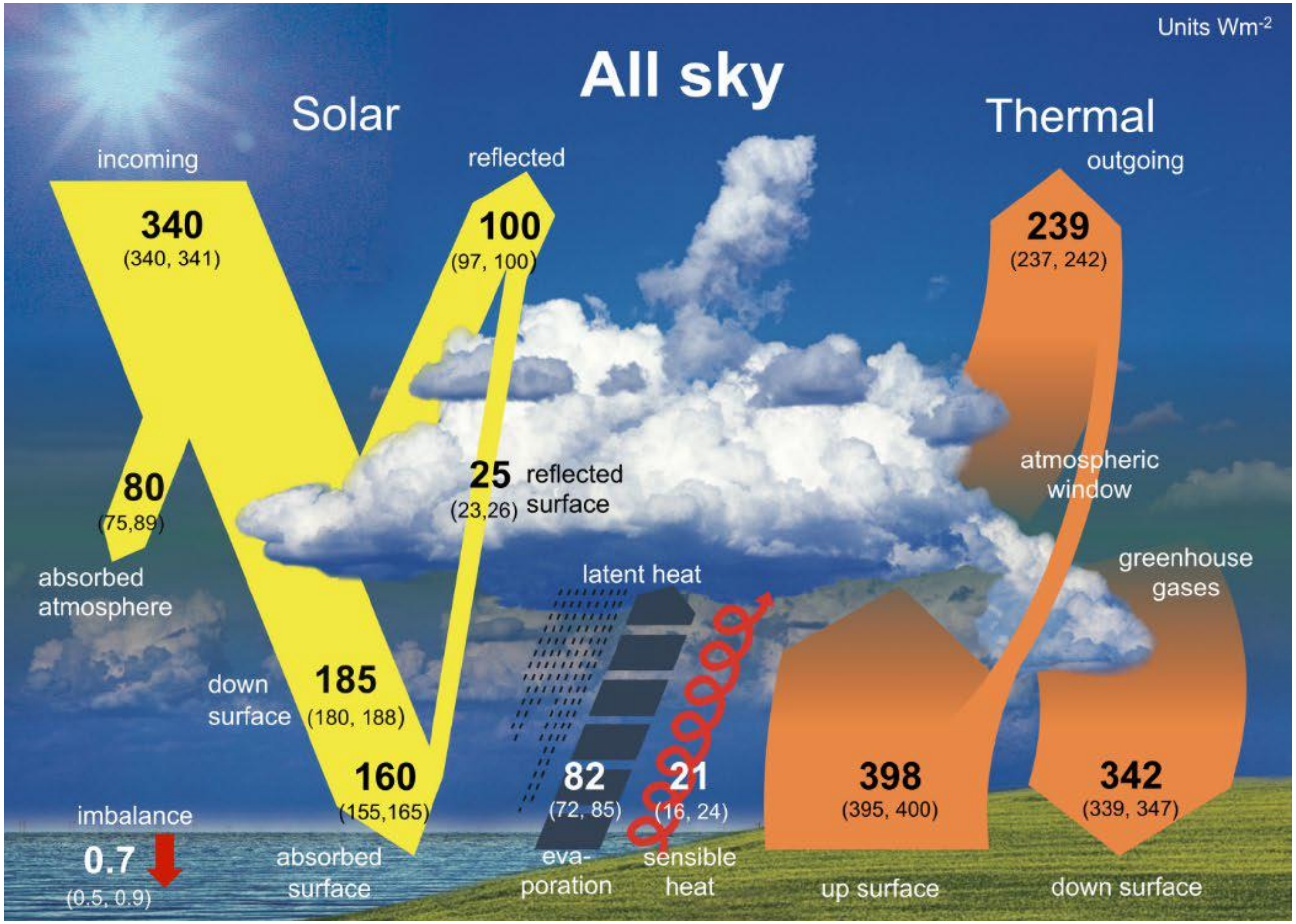
Outline

- Introduction
- Basics
- Absorption
- Scattering
- Radiative transfer
- Radiative equilibrium temperature
- Radiative heating and cooling

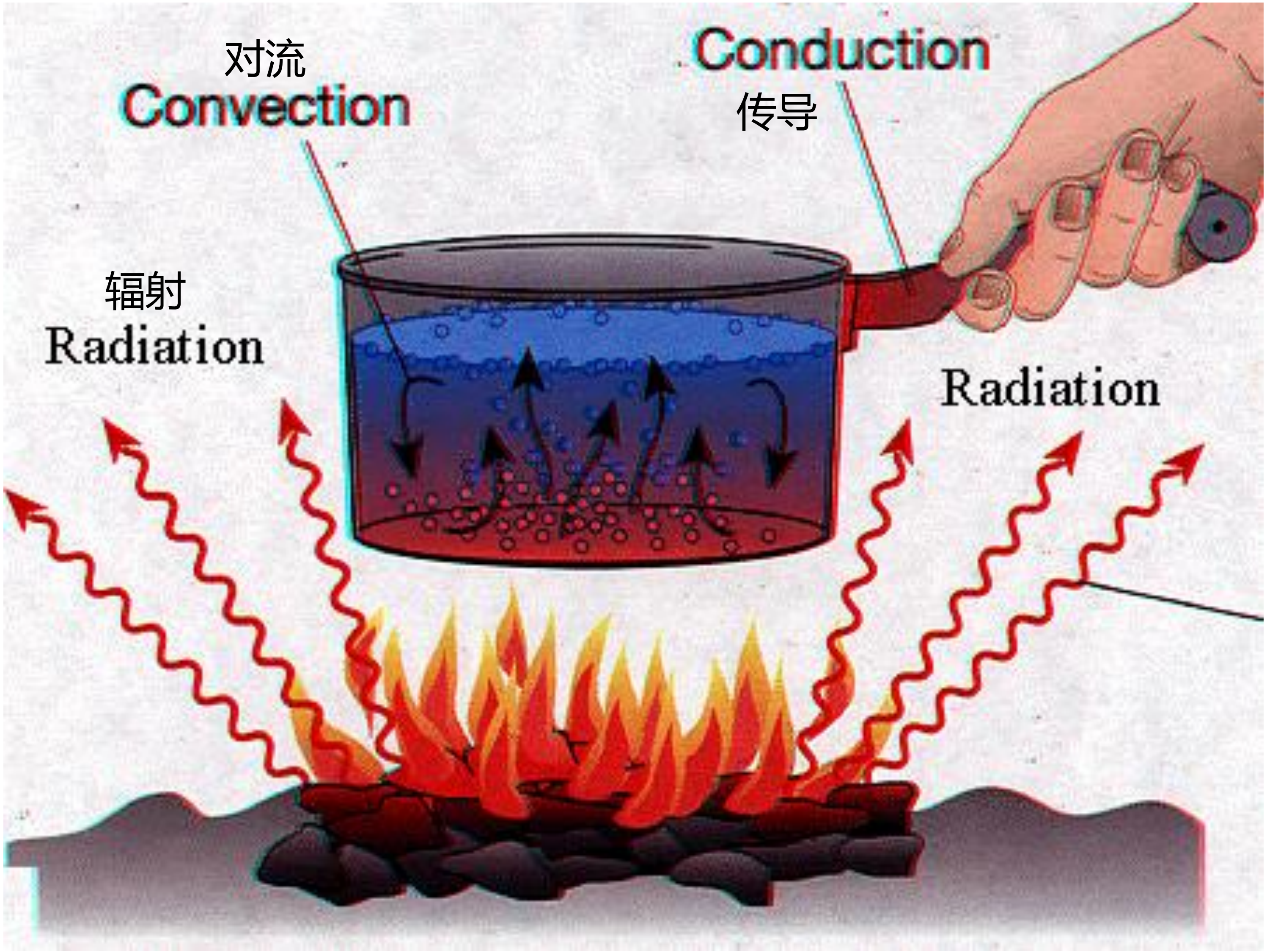
Earth Energy Budget (in Early 21st Century)



IPCC, 2021

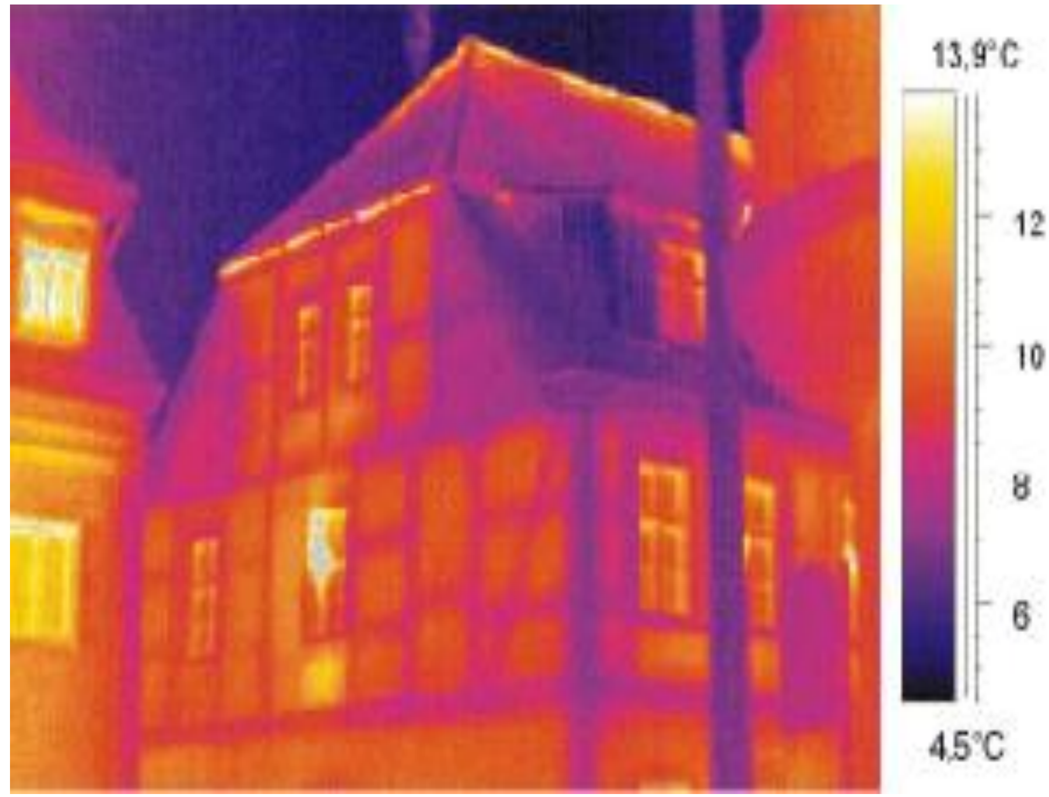
- Energy balance: Atmosphere 80+(398-40)+21+82-342-(239-40), Surface 160+342-398-21-82, Earth TOA 340-100-239
- Planetary albedo: ～29% (surface 7%, atmosphere 22%)

Radiation: An Effective Way of Energy Transfer



Thermal Radiation

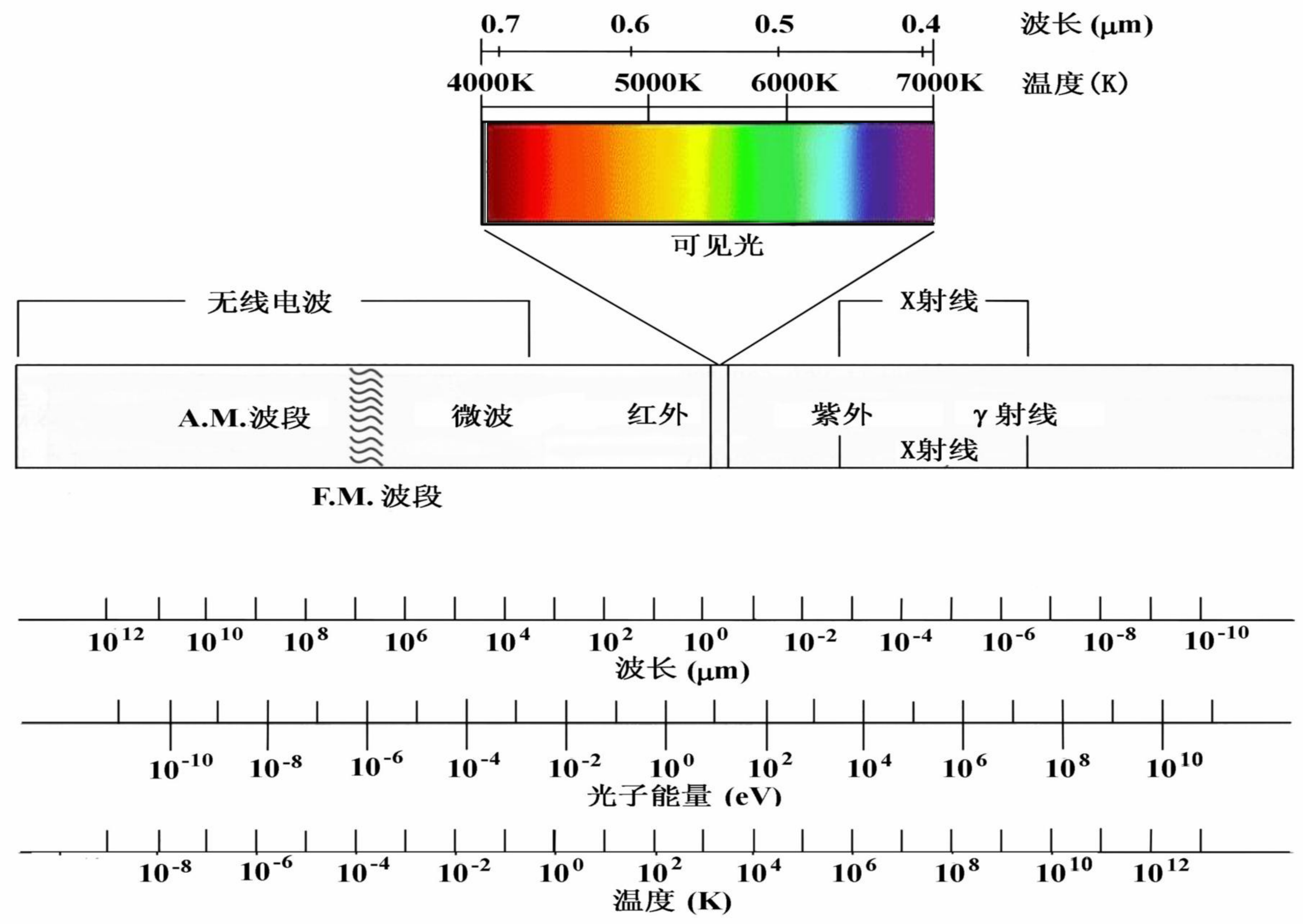
Thermal radiation is electromagnetic radiation emitted by the thermal motion of particles in matter



Solar radiative energy peaks at visible wavelength (~0.5 μm)

Terrestrial radiative energy peaks at ~10 μm, invisible by sole human eyes

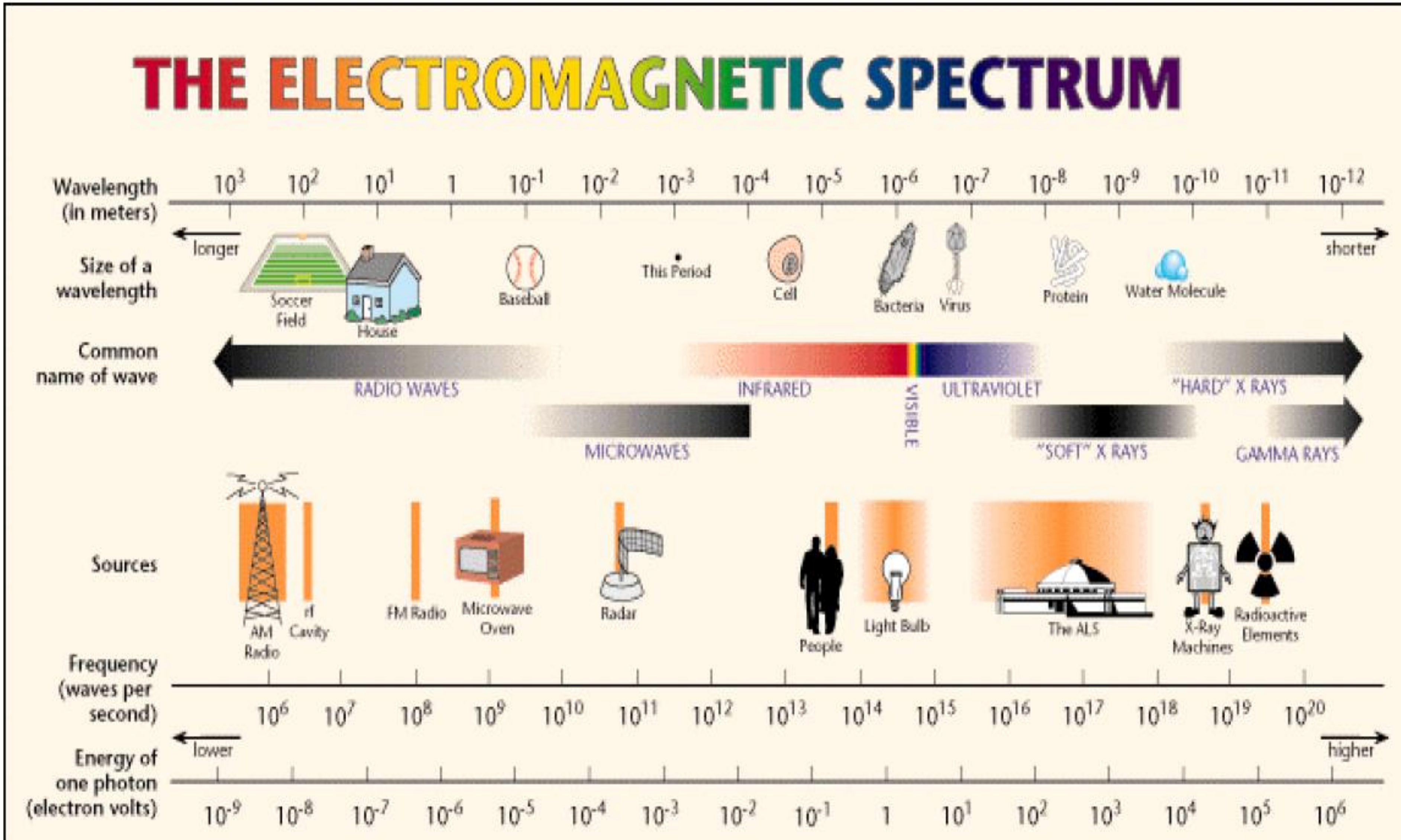
Electromagnetic Radiation Spectrum 电磁波谱



手机、微波炉：～1GHz，0.3m

$1\ eV = 1.6 \times 10^{-19}\ J$

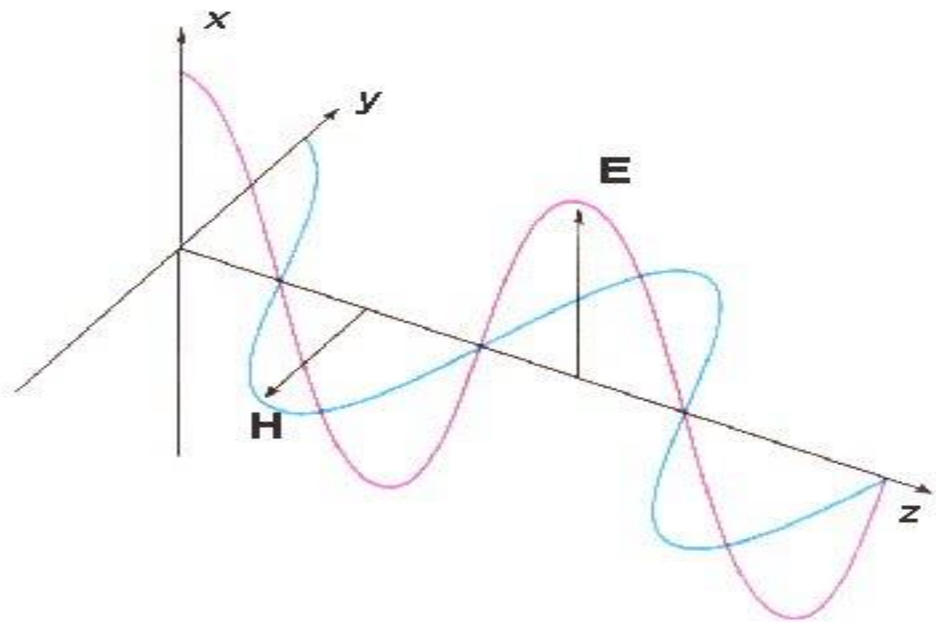
Electromagnetic Radiation and Sources



Electromagnetic Radiation Spectra in Atmospheric Sciences

Name of spectral region	Wavelength, µm	Spectral equivalence
Solar	0.1 – 4	= Shortwave = Ultraviolet + Visible + Near infrared
Terrestrial	4 – 100	= Longwave = Thermal = Far infrared
Ultraviolet	0.1 – 0.4	UV-A + UV-B + UV-C
Visible	0.4 – 0.7	
Infrared	0.7 – 100	= Near infrared + Far infrared
Near infrared	0.7 – 4	
Far infrared	4 – 100	
Microwave	$10^3 - 10^6$	
Radio	$> 10^6$	

Electromagnetic Radiation (EMR)



- EMR exhibits both wave properties and particulate properties.
- Self-propagating wave of the electromagnetic field that carries momentum and radiative energy through space.

Poynting Vector 能流密度适量

$$\boldsymbol{S} = \boldsymbol{E} \times \boldsymbol{H}$$

S is energy per unit time per unit area (e.g., W m^{-2})

E is electric field (unit: Volt/meter or Newton/Coulomb)

H is magnetic field (unit: ampere/meter)

In a propagating sinusoidal linearly polarized electromagnetic plane wave of a fixed frequency:

$$\langle \boldsymbol{S} \rangle = \frac{1}{\eta} \cdot |\boldsymbol{E_m}|^2$$

$\boldsymbol{E}_m$ is the complex amplitude of the electric field
η is the characteristic impedance of the transmission medium,
and $\eta_0 \approx 377\Omega$ for a plane wave in free space

Basic Concepts of Electromagnetic Radiation

电磁波可以用角频率(angular frequency, ω)、频率(frequency, f)、波长(wavelength, λ)、波数(wavenumber, υ)和波速(speed, V)来描述

$$\omega = 2\pi f$$

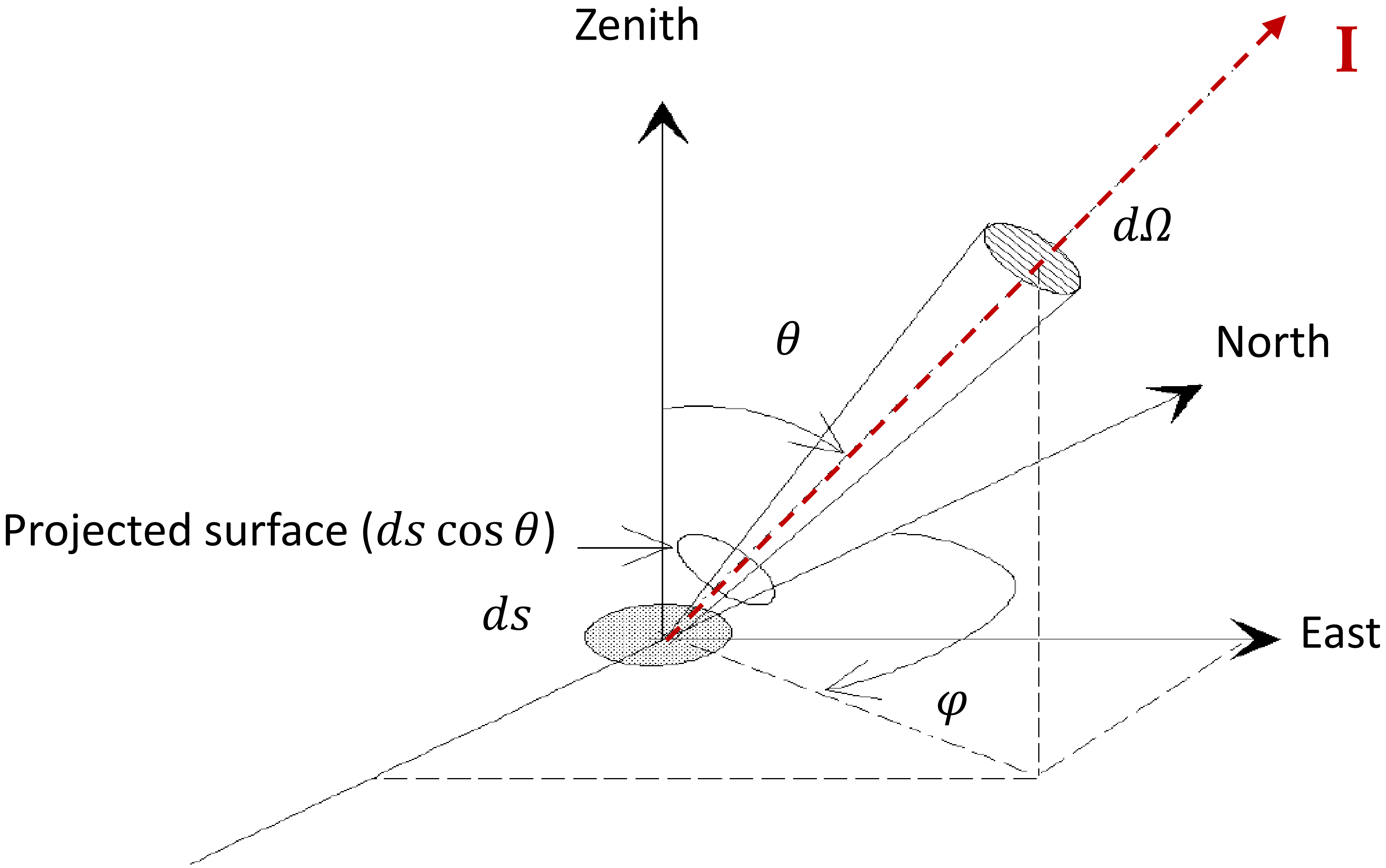
$$V = f\lambda \qquad df = ?\, d\lambda$$

$$\upsilon = 1/\lambda = f/V \qquad d\upsilon = ?\, d\lambda$$

波数变化1 cm^{-1}时，对应的波长变化：

- ✓ 在1000 cm^{-1} (10 μm) 处：10 nm
- ✓ 在10000 cm^{-1} (1 μm) 处：0.1 nm

Radiance or Intensity: （分光）辐亮度



Radiance or Intensity: （分光）辐亮度

- **Radiance is monochromatic energy per unit time per unit area per solid angle**

$$I = dQ/dt/dA/d\lambda/d\Omega = dQ/dt/dA/d\lambda/(\sin\theta\, d\theta d\varphi)$$

Unit: $w/m^2/\mu m/sr$, where sr = steradian 立体弧度

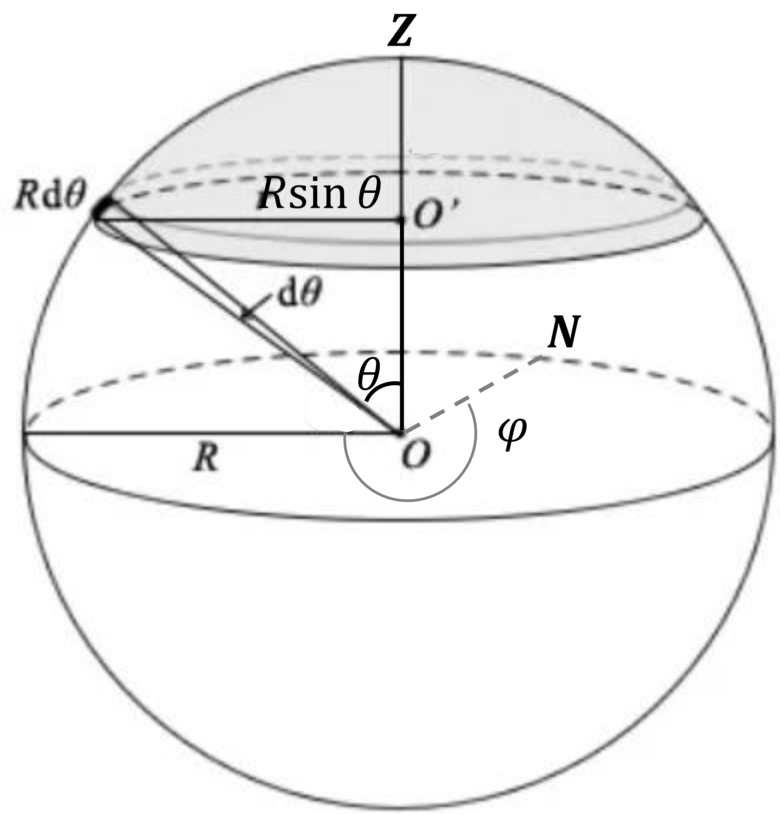
Ω = Solid angle 立体角, $d\Omega = \sin\theta\, d\theta d\varphi$

- **Radiance is a 7-dimensional variable**

$$I(t; x, y, z; \lambda; \Omega) = I(t; x, y, z; \lambda; \theta, \varphi)$$

- **Radiance can be measured by photometer 光度计**

Calculation of Solid Angle



球面的面积元

$$dS = R^2 d\Omega = R^2 \sin\theta \, d\theta d\varphi$$

球表面积

$$S = \oint_{\Omega} dS = \int_{\theta=0}^{\pi}\int_{\varphi=0}^{2\pi} R\cdot\sin\theta\, d\varphi\cdot R\, d\theta = 4\pi R^2$$

全空间立体角

$$\oint_{\Omega} d\Omega = \int_{\theta=0}^{\pi}\int_{\varphi=0}^{2\pi} \sin\theta\, d\varphi\cdot d\theta = 4\pi$$

Radiance versus Flux Density

- **Flux Density or Irradiance（分光）辐射通量密度:**

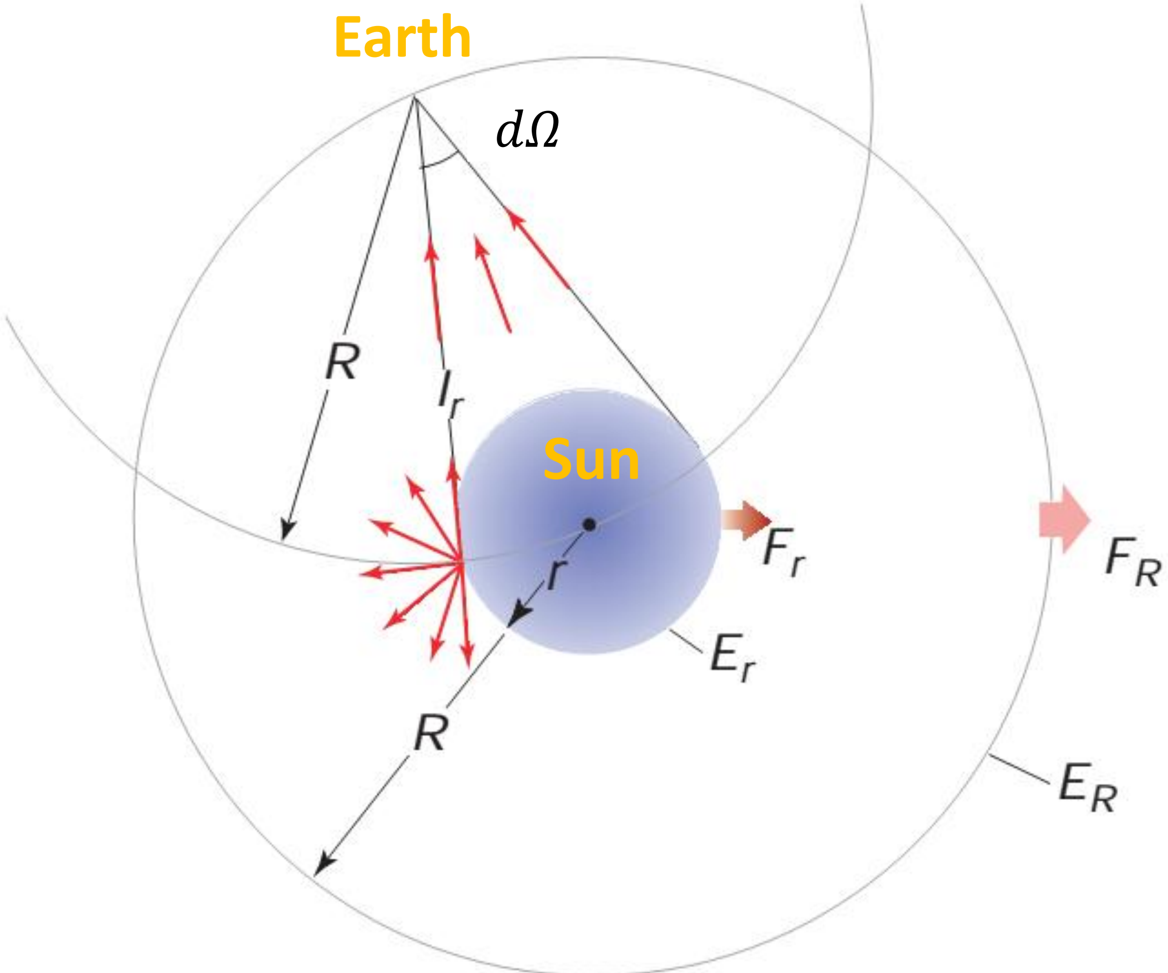
$$F_\lambda = \int_\Omega I_\lambda \cdot \cos\theta \cdot d\Omega = \int_\theta \int_\varphi I_\lambda \cdot \cos\theta \cdot \sin\theta \, d\varphi \cdot d\theta$$

- **Within a range of wavelength:**

$$F_{\lambda_1,\lambda_2} = \int_{\lambda_1}^{\lambda_2} F_\lambda \cdot d\lambda$$

平行光?

Solar Radiance and Irradiance



Integrating all wavelengths, then $I_s = 2.00 \times 10^7$ W m^{-2} sr^{-1}

✓ Given R, r, and F_R, use two methods to calculate I_s ?

（电磁）辐射源，点源和面源

- 往外发射辐射的物体称为辐射源。
- 最简单的辐射源是点源。这是一种理想的情况，即其几何尺度可以被忽略。
- 面辐射源向外（2π 立体角）发射辐射。我们绝大部分时间遇到的都是这种源。对面辐射源首先关心的是其出射的辐射通量密度（辐出度），即单位时间内通过单位面积在面的法线方向射出的能量有多少。

Lambertian Surface（朗伯面）

在大气科学中，我们常常用到朗伯面，其定义是该表面向所有 2π 立体角方向发出（作为光源）或者反射出（作为反射物）均一的辐亮度 *I*

$$F = \int_0^{2\pi} I \cos\theta \, d\Omega = I \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \cos\theta \sin\theta \, d\theta d\varphi = \pi I$$

吸收、反射、透射

- 入射至物体的辐射能，一部分会被物体吸收变为物体的内能或其它形式的能量，一部份会被反射回去，而另一部分则会透过物体。

- 投射到物体的辐射能为Q_0，其中部分被吸收Q_a、部分被反射Q_r、部分被透射Q_t。根据能量守恒：

$$Q_0 = Q_a + Q_r + Q_t$$

- 定义：吸收率$A = Q_a / Q_0$，反射率$R = Q_r / Q_0$，透射率$T = Q_t / Q_0$，则：

$$A + R + T = 1$$

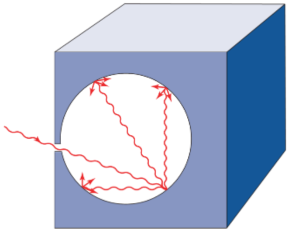
- A、R、T 与波长有关！

黑体和灰体

- 绝对黑体（blackbody）：对任何波长的辐射都能全部吸收，即$A = 1$。相应的必有$R = 0$，$T = 0$。
- 绝对黑体在自然界几乎是不存在的（黑洞？），但在实验室可以人工制造出尽可能接近于绝对黑体的物质。
- 如果物体仅对特定波长范围有$A_l = 1$，则称其在该波长范围为黑体。

【2014年，英国萨里纳米系统公司Surrey Nanosystems推出了一种全新材料Vantablack，对于750nm波长的吸收率达到创纪录的99.965%】

- 如果物体的吸收率A（在一定波长范围内）不随波长而变，但$A < 1$，则称该物体为对应的灰体。
- 黑体 v.s. 黑色物体？ 冰雪？



热力学平衡、热平衡、温度

- 热力学平衡：热平衡、力平衡、辐射平衡、化学平衡
- 孤立系统的热平衡：孤立系统内部无净热交换。可以用一态函数“温度”来描述。
- 局地（准）热平衡：局地孤立系统内部达到热平衡的时间远小于与外界作用的时间，因此系统内部有一个近似一致的温度。这一温度可以受到外界影响而随时间变化。
- 对流层和平流层大气可视为处于局地（准）热平衡状态，因此可以应用热平衡辐射的规律来研究其大气辐射问题。
- 如何理解“全球平均温度”、“体温”？

基尔霍夫（热辐射）定律

- 基尔霍夫在1859年由热力学定律论证指出：

  在一定的温度 T 时，任何处于热力学平衡的物体向外发射的辐亮度 $I_{\lambda,T}$ 和它的吸收率 $A_{\lambda,T}$ 之比值是一个普适函数 $B(\lambda,T)$ 。

- $B(\lambda,T)$ 就是绝对黑体的辐亮度，它只是温度和波长的函数，而与物体的其他性质无直接关系。

$$\frac{I_{\lambda,T}}{A_{\lambda,T}} = B(\lambda,T) \qquad \varepsilon(\lambda,T) = \frac{I_{\lambda,T}}{B(\lambda,T)} = A_{\lambda,T}$$

发射率 = 吸收率

普朗克定律（绝对黑体辐射）

1900年，普朗克引入量子概念，得到：

$$B(\lambda,T)=\frac{2c^2h}{\lambda^5}\left(e^{\frac{ch}{k\lambda T}}-1\right)^{-1}=\frac{c_1}{\lambda^5}\left(e^{\frac{c_2}{\lambda T}}-1\right)^{-1}$$

$$c_1=2c^2h=1.19\times10^8\ W\ \mu m^4\ m^{-2}\ sr^{-1}$$

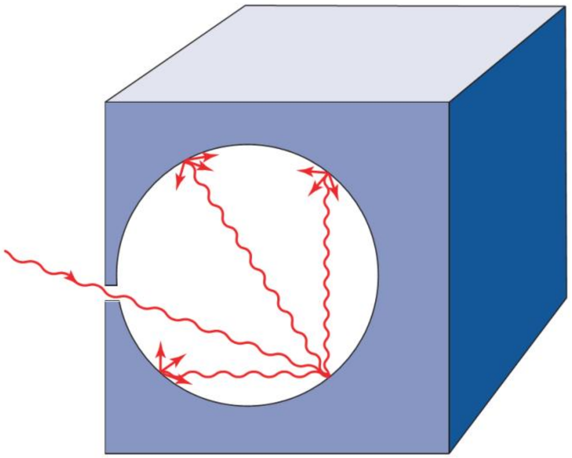
$$c_2=\frac{ch}{k}=14388\ \mu m\ K$$

B (λ, T) 的单位为 W m^{-2} μm^{-1} sr^{-1}

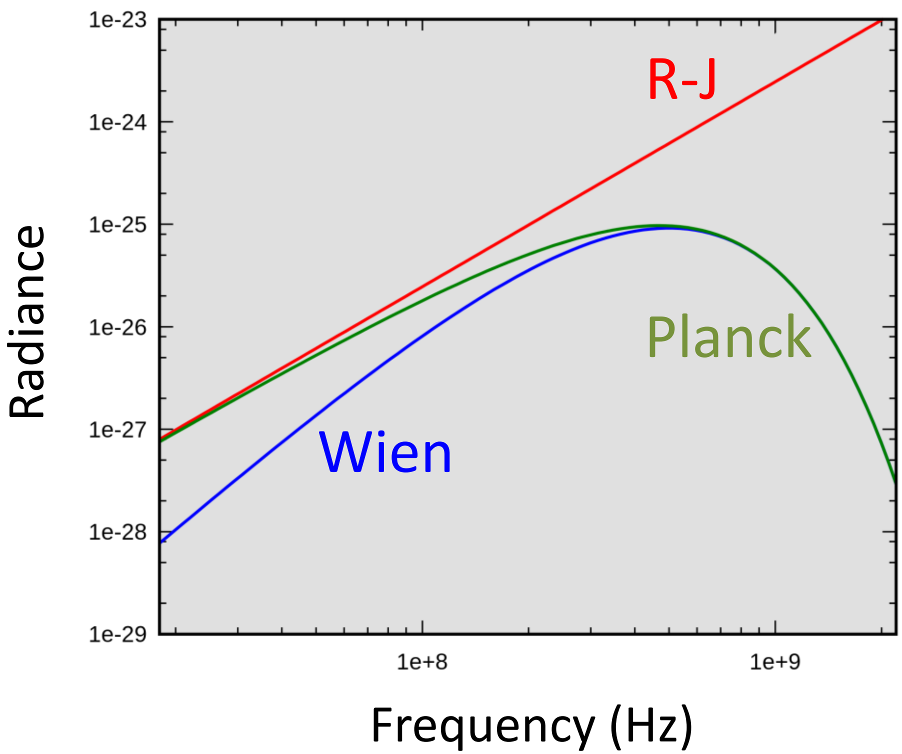
c = 2.99793×10^{14} μm s^{-1}，是真空光速

h = 6.6262×10^{-34} J s，是普朗克常数

k =1.3806×10^{-23} J K^{-1}，是波尔兹曼常数



普朗克定律、瑞利-金斯近似、维恩近似



长波 → 短波

Planck:

$$B(\lambda, T) = \frac{2c^2h}{\lambda^5}\left(e^{\frac{ch}{k\lambda T}} - 1\right)^{-1}$$

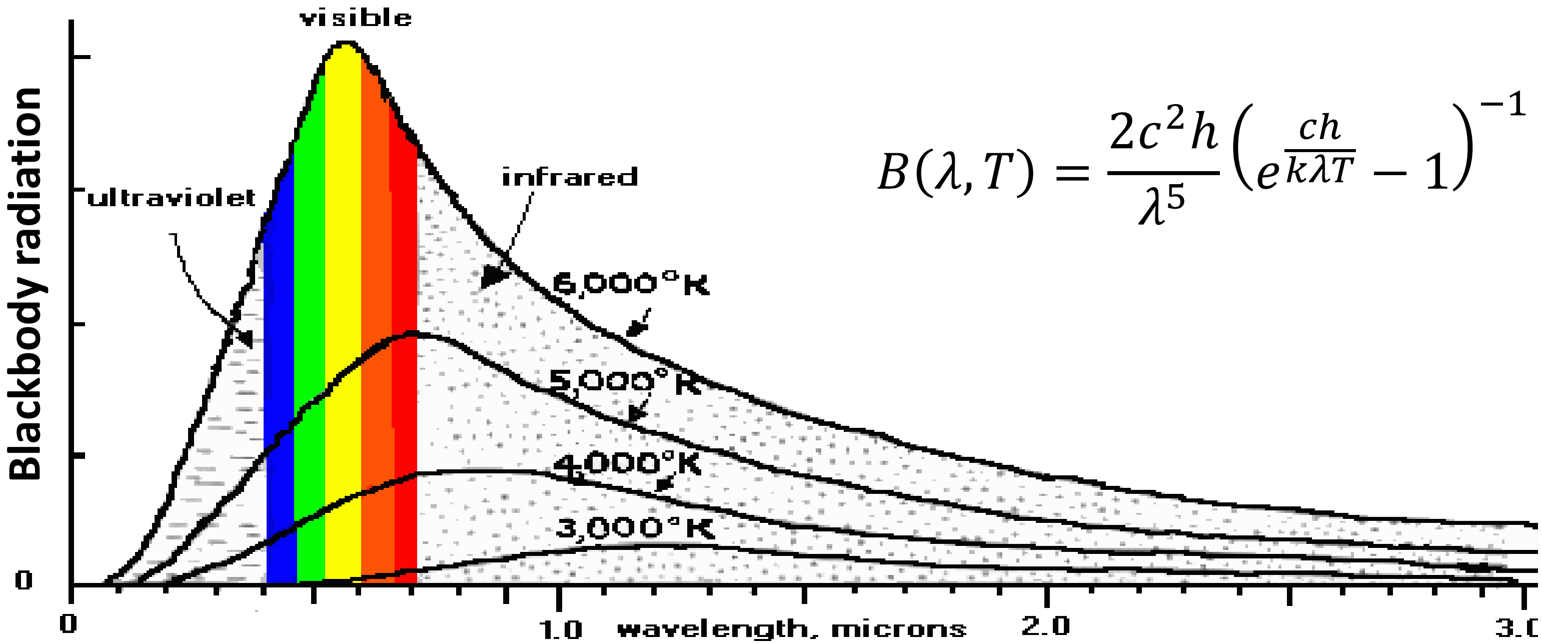
R-J (1905): $$B(\lambda, T) = \frac{2ckT}{\lambda^4}$$

λ → ∞

Wien (1896): $$B(\lambda, T) = \frac{2c^2h}{\lambda^5} e^{-\frac{ch}{k\lambda T}}$$

λ → 0

Blackbody Radiation



$$B(\lambda, T) = \frac{2c^2 h}{\lambda^5}\left(e^{\frac{ch}{k\lambda T}} - 1\right)^{-1}$$

维恩位移定律(1879年)

$$\lambda_{max} = a/T$$

a = 2897.8 μm K

斯蒂芬–玻尔兹曼定律（1893年）

对所有波长积分:

$$F = \pi B(T) = \sigma T^4$$

$\sigma = 5.67\times10^{-8}$ W m^{-2} K^{-4}

斯蒂芬–玻尔兹曼常数

Bose–Einstein integral

$$\int_0^\infty \frac{x^3}{e^x - 1}dx = \frac{\pi^4}{15}$$

$$\sigma = \frac{2k^4\pi^5}{15c^2h^3}$$

Blackbody Radiation: Cumulative Energy Distribution

$$B(\lambda, T) = \frac{2c^2 h}{\lambda^5}\left(e^{\frac{ch}{k\lambda T}} - 1\right)^{-1} = \frac{c_1}{\lambda^5}\left(e^{\frac{c_2}{\lambda T}} - 1\right)^{-1}$$

Percentile	0.01%	0.1%	1%	10%	20%	**25.0%**	30%	40%	**41.8%**	50%	60%	**64.6%**	70%	80%	90%	99%	99.9%	99.99%
λT (µm·K)	910	1110	1448	2195	2676	**2898**	3119	3582	**3670**	4107	4745	**5099**	5590	6864	9376	22884	51613	113374

Percentile	0.01%	0.1%	1%	10%	20%	**25.0%**	30%	40%	**41.8%**	50%	60%	**64.6%**	70%	80%	90%	99%	99.9%	99.99%
Sun λ (nm)	157	192	251	380	463	**502**	540	620	**635**	711	821	**882**	967	1188	1623	3961	8933	19620
288 K planet λ (µm)	3.16	3.85	5.03	7.62	9.29	**10.1**	10.8	12.4	**12.7**	14.3	16.5	**17.7**	19.4	23.8	32.6	79.5	179	394

At which wavelength the blackbody radiation per unit of wavelength peaks?
At which wavelength the blackbody radiation per unit of frequency peaks?
At which wavelength the blackbody radiation per unit of wave number peaks?

思考题

普朗克定律可以以波长 λ、频率 f 或者波数 ν 形式来表达：

$$B(\lambda,T)=\frac{2c^2h}{\lambda^5}\left(e^{\frac{ch}{k\lambda T}}-1\right)^{-1}$$

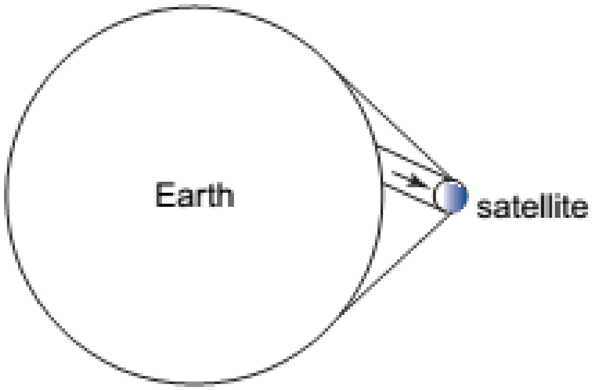
$$B(f,T)=\frac{2f^3h}{c^2}\left(e^{\frac{fh}{kT}}-1\right)^{-1}$$

$$B(\nu,T)=2\nu^3c^2h\left(e^{\frac{\nu ch}{kT}}-1\right)^{-1}$$

A. 从波长形式推导其他两种形式

B. 当普朗克函数用波长、频率、波数表达时，都会得到一个极值点。若把极值点对应的波长、频率、波数都转换成对应的波长，这3个波长是否相等？

思考题



A. A small, perfectly black, spherical satellite is in orbit around the Earth at an altitude of 2000 km. What angle does the Earth subtend when viewed from the satellite?

B. If the Earth radiates as a blackbody at an equivalent blackbody temperature Te = 255 K, calculate the radiative equilibrium temperature of the satellite when it is in the Earth's shadow.