Radiation

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致谢：本课件中部分资料来自李成才老师（特别是关于辐射的部分）。

Outline

- Introduction
- Concepts
- Absorption
- Scattering
- Radiative transfer
- Radiative equilibrium temperature
- Radiative heating and cooling

思考题

Consider thermal emission from an isothermal, nonblack cloud. Suppose there is a stratus cloud with stratified atmosphere. The top of the cloud is at $\tau = 0$, and the bottom is at $\tau = \infty$. Assume that we are in the thermal region ($F_{\boxed{*}} = 0$), that the cloud is isothermal ($\frac{dB}{d\tau} = 0$), that the scattering is isotropic ($g = 0$), and that the single scattering albedo (ω_0) is a constant. Thus, the radiative transfer of diffuse flux becomes:

$$\frac{d^2F}{d\tau^2} = \alpha^2 F, \qquad \alpha^2 = 3(1 - \omega_0)$$

(i) Show that the boundary condition at TOA ($\tau = 0$) is:

$$\left(\frac{dF}{d\tau}\right)_{\tau=0} = 4\pi(1 - \omega_0)\left(\frac{F(0)}{2\pi} - B\right)$$

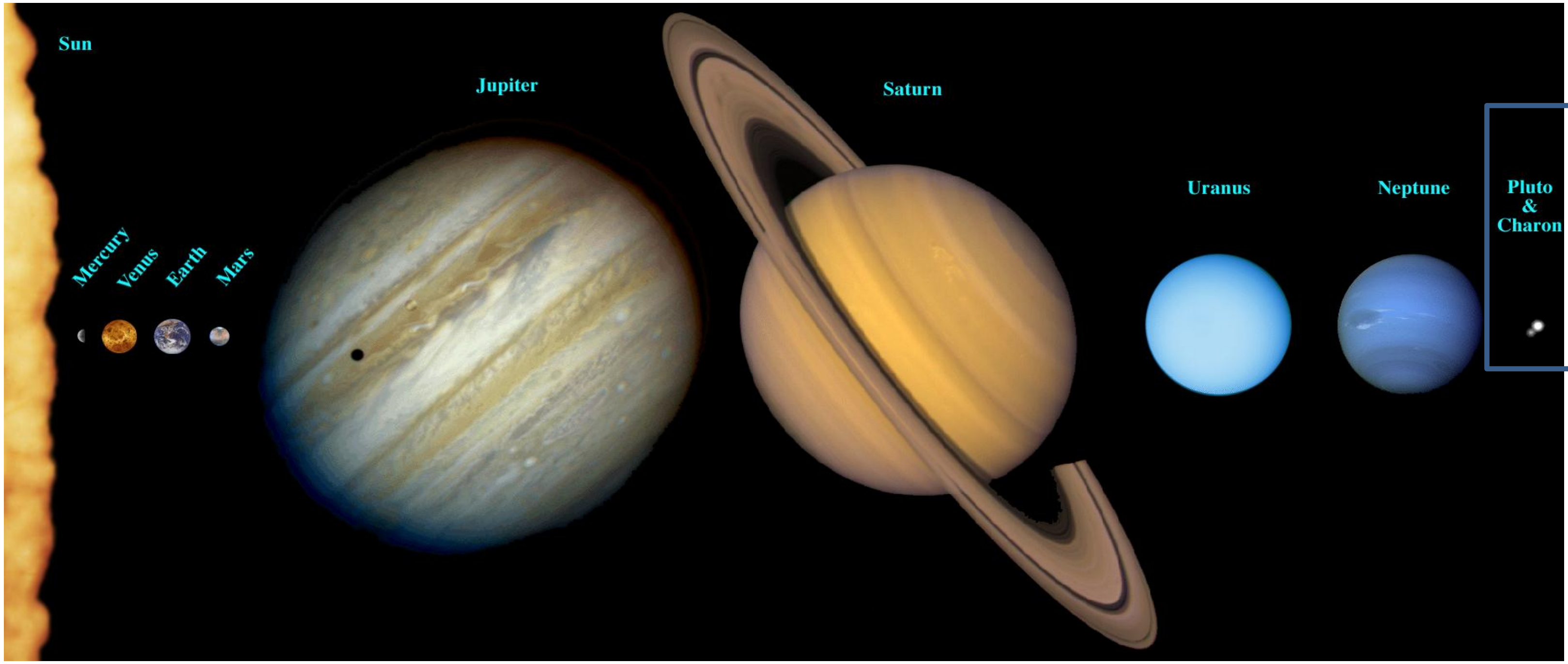
and:

$$\frac{F(0)}{\pi B} = \frac{4(1-\omega_0)}{2(1-\omega_0)+\sqrt{3(1-\omega_0)}}$$

(ii) The above formulae are approximate. What should be flux be for a black body ($\omega_0 = 0$)? Correct the above formula with a constant factor to allow for this error and calculate $\frac{F(0)}{\pi B}$ for $\omega_0 = 1.0, 0.8, 0.6, 0.4, 0.2, 0.0$.

Richard Goody

Planets in the Solar System



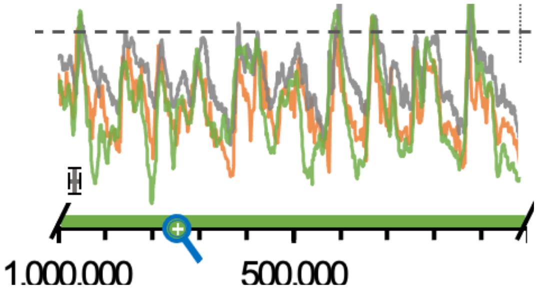
dwarf planet

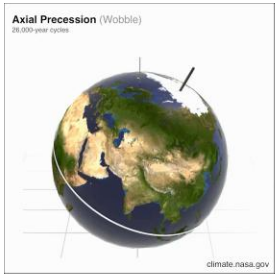
- 太阳活动？
- 地球公转轨道？
- 黄赤交角？
- 岁差？
- 可宜居性？

地球基本参数

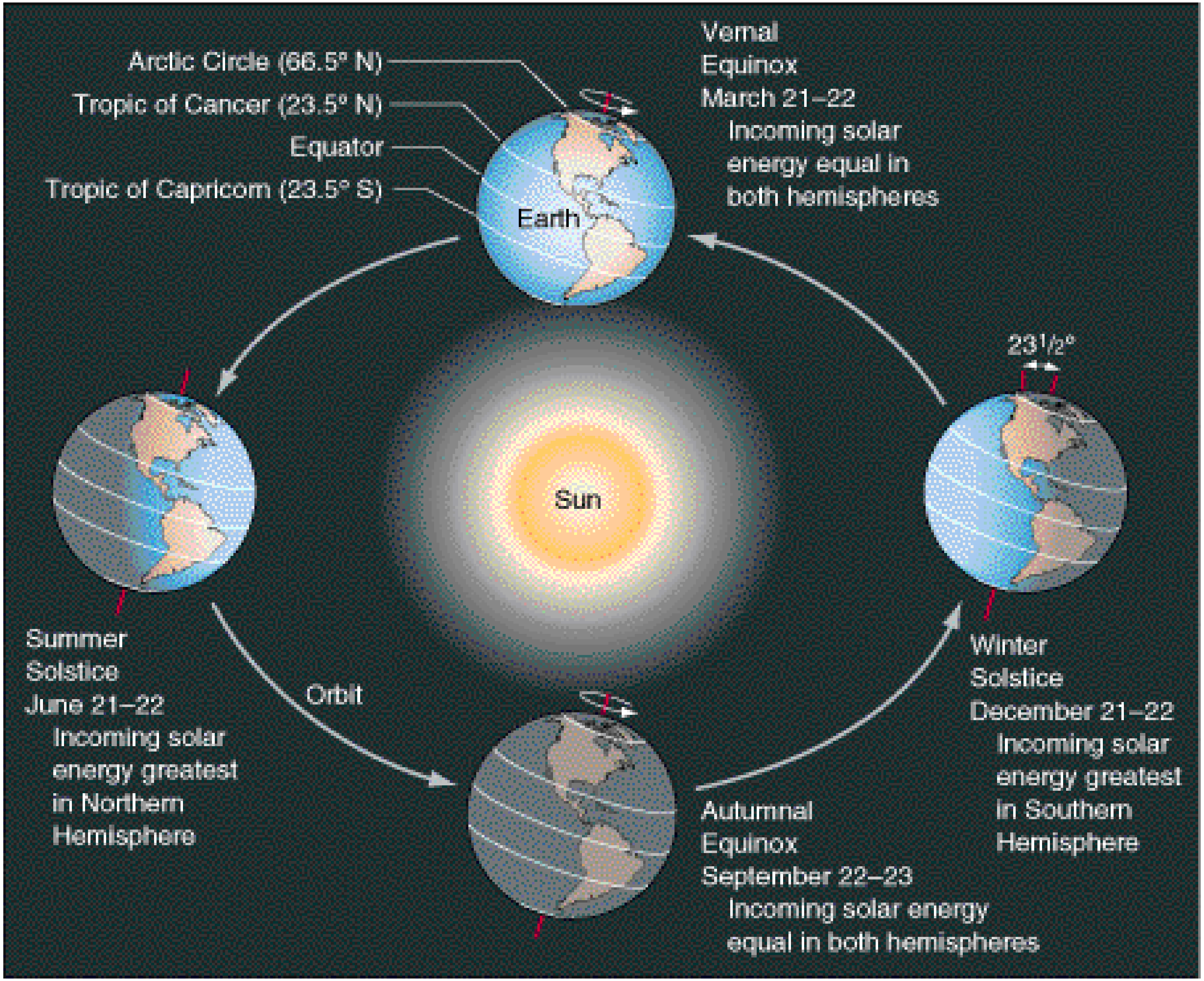
- 日地距离: 平均1.496e8 km, or 1 AU （range：0.973–1.017 AU）
- 地球轨道偏心率：0.0167（周期：40万年、10万年）
- 地球自转轴角度：～23.5º（周期：4.1万年）
- 地球自转轴进动（岁差；周期：2.6万年）
- 地球公转周期：1年，自转周期：1天
- 地球半径：～6371 km（极半径 6357 km，赤道半径 6378 km）
- 太阳常数：日地平均距离处的太阳辐射，1361–1362 w/m^2
- 地球系统的太阳辐射返照率：～29%，太阳辐射量净量：～240 w/m^2

米兰科维奇理论？

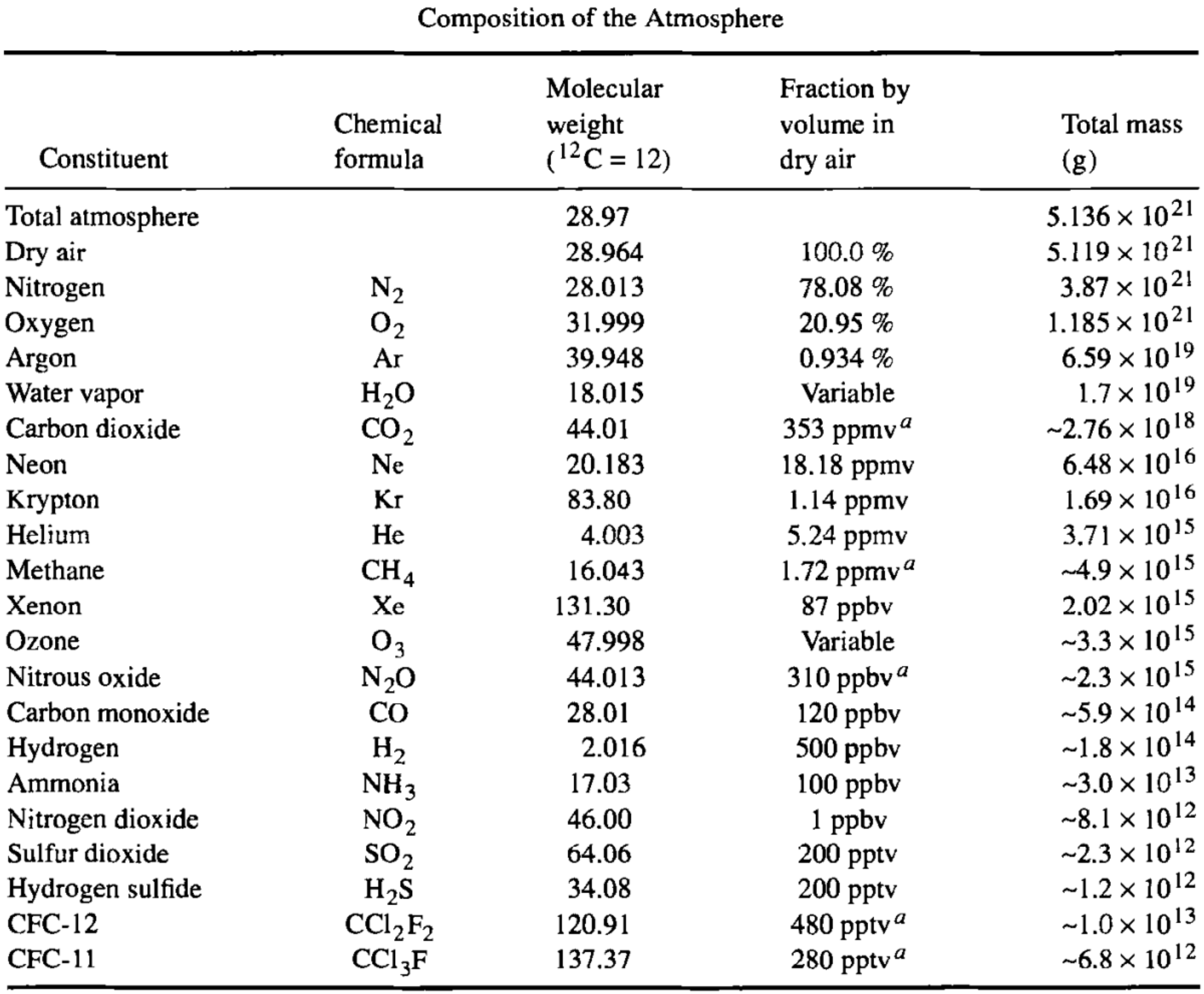




Seasons


Arctic Circle (66.5° N)
Tropic of Cancer (23.5° N)
Equator
Tropic of Capricorn (23.5° S)
Earth
Vernal Equinox March 21–22 Incoming solar energy equal in both hemispheres
23½°
Sun
Summer Solstice June 21–22 Incoming solar energy greatest in Northern Hemisphere
Orbit
Winter Solstice December 21–22 Incoming solar energy greatest in Southern Hemisphere
Autumnal Equinox September 22–23 Incoming solar energy equal in both hemispheres

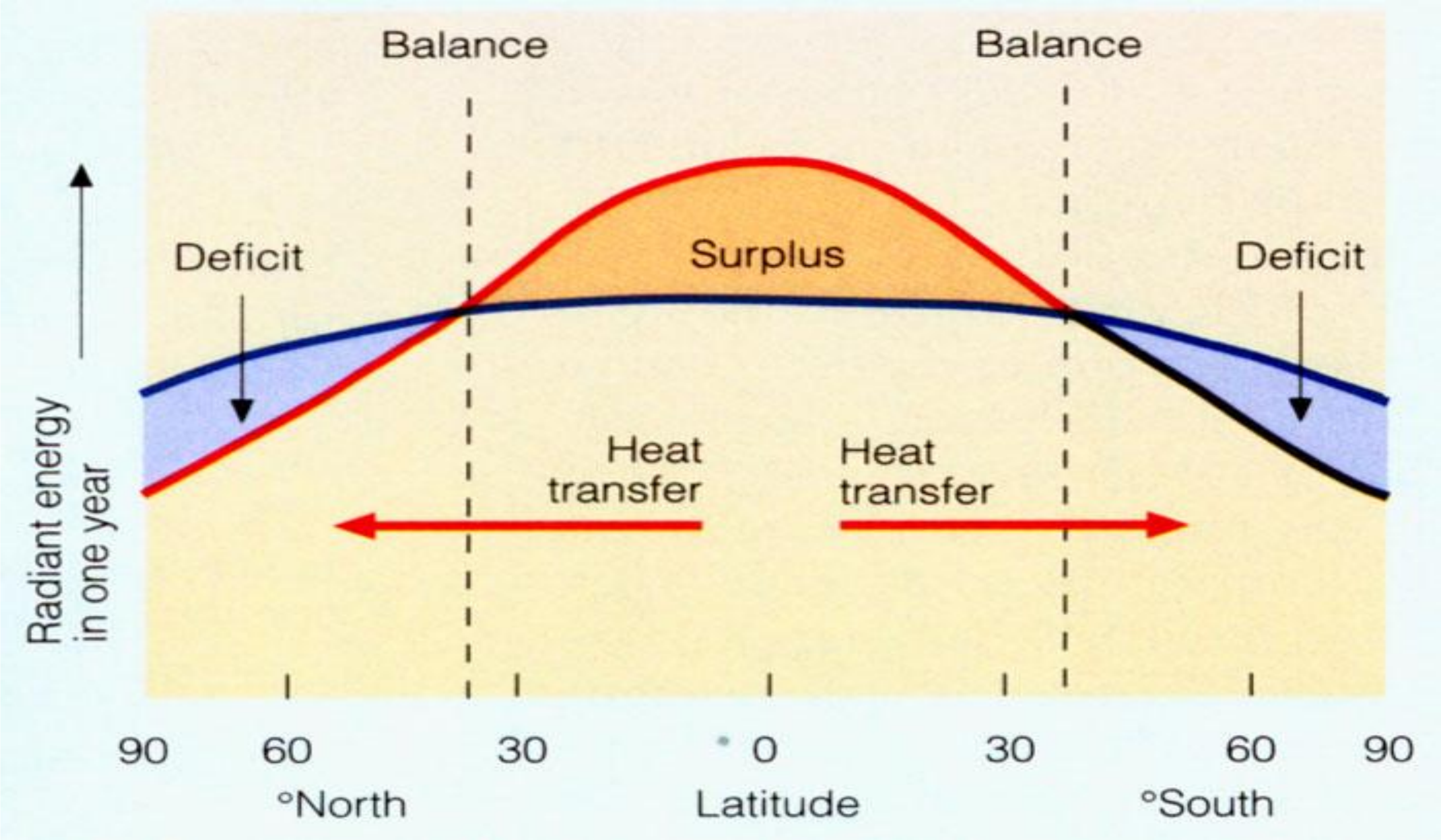
Atmospheric Compositions



Composition of the Atmosphere

Constituent	Chemical formula	Molecular weight ($^{12}C = 12$)	Fraction by volume in dry air	Total mass (g)
Total atmosphere		28.97		5.136×10^{21}
Dry air		28.964	100.0 %	5.119×10^{21}
Nitrogen	N_2	28.013	78.08 %	3.87×10^{21}
Oxygen	O_2	31.999	20.95 %	1.185×10^{21}
Argon	Ar	39.948	0.934 %	6.59×10^{19}
Water vapor	H_2O	18.015	Variable	1.7×10^{19}
Carbon dioxide	CO_2	44.01	353 ppmv[a]	$\sim 2.76 \times 10^{18}$
Neon	Ne	20.183	18.18 ppmv	6.48×10^{16}
Krypton	Kr	83.80	1.14 ppmv	1.69×10^{16}
Helium	He	4.003	5.24 ppmv	3.71×10^{15}
Methane	CH_4	16.043	1.72 ppmv[a]	$\sim 4.9 \times 10^{15}$
Xenon	Xe	131.30	87 ppbv	2.02×10^{15}
Ozone	O_3	47.998	Variable	$\sim 3.3 \times 10^{15}$
Nitrous oxide	N_2O	44.013	310 ppbv[a]	$\sim 2.3 \times 10^{15}$
Carbon monoxide	CO	28.01	120 ppbv	$\sim 5.9 \times 10^{14}$
Hydrogen	H_2	2.016	500 ppbv	$\sim 1.8 \times 10^{14}$
Ammonia	NH_3	17.03	100 ppbv	$\sim 3.0 \times 10^{13}$
Nitrogen dioxide	NO_2	46.00	1 ppbv	$\sim 8.1 \times 10^{12}$
Sulfur dioxide	SO_2	64.06	200 pptv	$\sim 2.3 \times 10^{12}$
Hydrogen sulfide	H_2S	34.08	200 pptv	$\sim 1.2 \times 10^{12}$
CFC-12	CCl_2F_2	120.91	480 pptv[a]	$\sim 1.0 \times 10^{13}$
CFC-11	CCl_3F	137.37	280 pptv[a]	$\sim 6.8 \times 10^{12}$

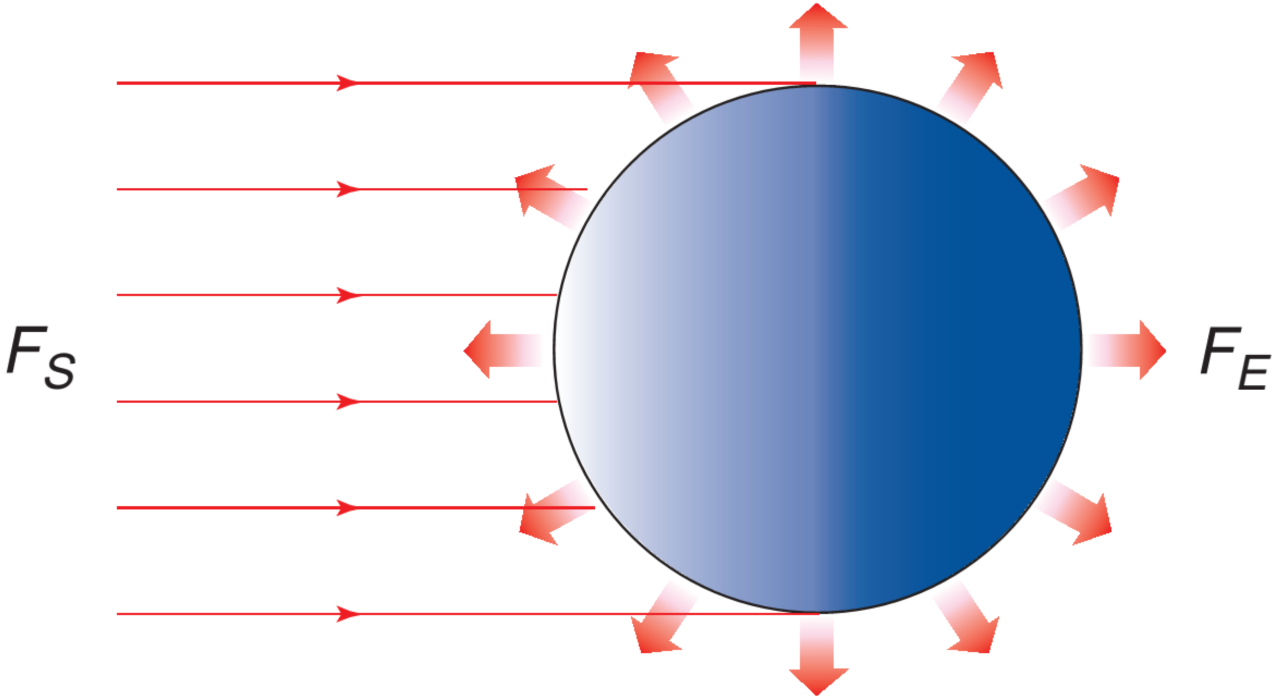
Longwave and Shortwave Radiation Imbalance



地气系统辐射平衡：一箱模型

- 从全球长期平均温度来看，地气系统的温度多年基本不变，所以全球应当是（基本）达到辐射平衡的。
- 可以用一个简单的地气系统辐射平衡模式来估算地气系统的辐射平衡。

$F_S = \mathrm{F}_E$ （辐射通量，w）



地气系统辐射平衡

- 设地球系统是一个半径为 r 的球，它对太阳短波辐射的反射率（行星反照率）为 R。则其接收的短波辐射通量

$$F_s = S_0 \pi r^2 (1 - R)$$

其中，S_0 为太阳常数

- 地气系统（在长波处）可看作为黑体，其等效温度为 T_e，它向宇宙空间发射的长波辐射通量

$$F_E = 4\pi r^2 \sigma T_e^4$$

地气系统辐射平衡

- 在地气系统达到辐射平衡时

$$S_0\pi r^2(1-R)=4\pi r^2\sigma T_e^4$$

因此，$T_e=\sqrt[4]{\frac{S_0(1-R)}{4\sigma}}$

- 取 $S_0=1361\,w/m^2$，$R=0.29$，可算出

$$T_e=\ 255.5\,K\ \ \approx -18\,℃$$

T_e 为地气系统能量平衡时的“等效温度”

这比观测到的地表平均温度(288 K)低了33 K！

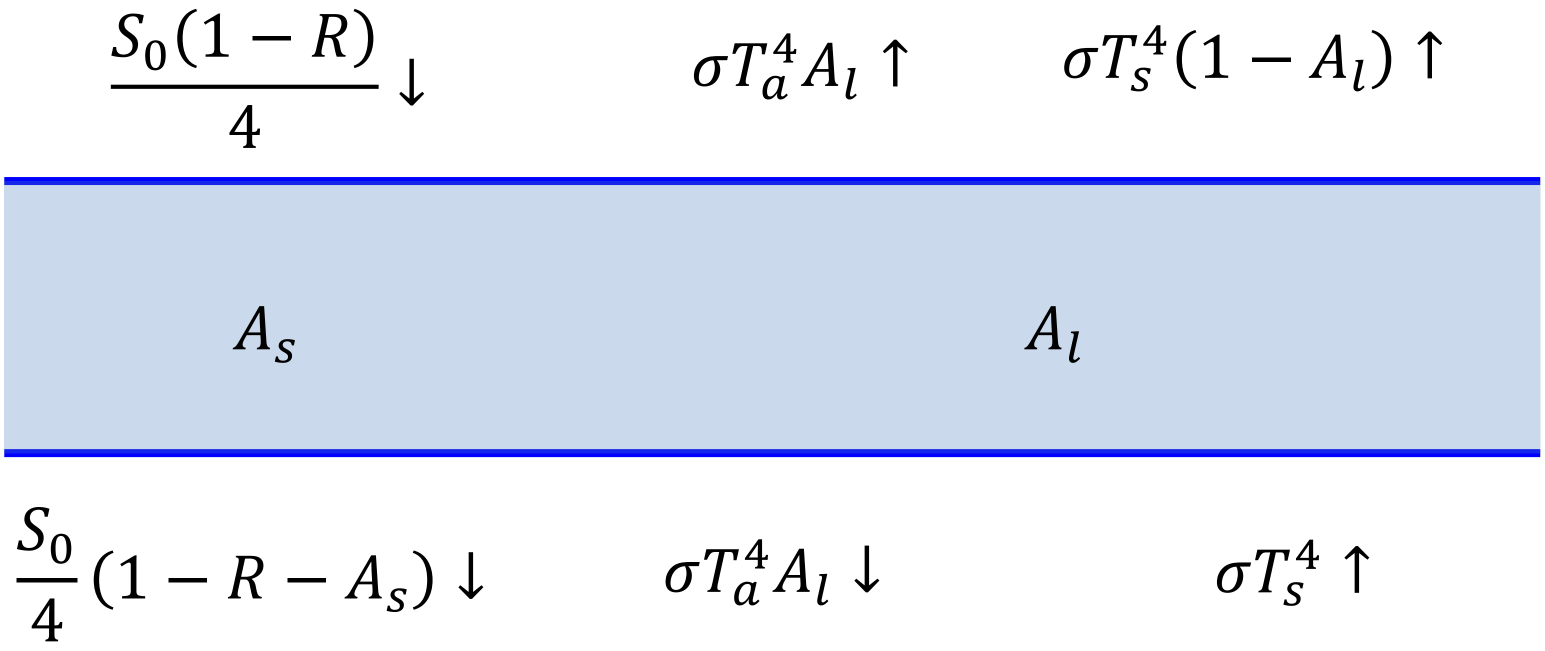
太阳系行星气候

行星	离太阳距离 (10^6km)	太阳常数 (w m^{-2})	*R*=0.29 时的*Te* (K)	实测的行星反照率	实测 *R* 下的*Te* (K)
水星	58			0.06	442
金星	108			0.78	227
地球	150	1361		0.3	255
火星	228			0.17	216
木星	778			0.45	106

“大气+地表”辐射平衡：两箱模型

利用“大气+地表”辐射平衡，计算地面平衡温度：

其中，R为地气系统对短波辐射的返照率，A_s为大气短波(净)吸收率，A_l为大气长波(净)吸收率，地面在长波波段为黑体



$$\frac{S_0(1-R)}{4}\downarrow \qquad \sigma T_a^4 A_l \uparrow \qquad \sigma T_s^4(1-A_l)\uparrow$$

$$A_s \qquad A_l$$

$$\frac{S_0}{4}(1-R-A_s)\downarrow \qquad \sigma T_a^4 A_l \downarrow \qquad \sigma T_s^4 \uparrow$$

“大气+地表”辐射平衡

TOA辐射平衡： $\frac{S_0}{4}(1-R)=\sigma T_a^4 A_l+\sigma T_s^4(1-A_l)$

地表辐射平衡： $\frac{S_0}{4}(1-R-A_s)+\sigma T_a^4 A_l=\sigma T_s^4$

因此，

地表温度： $T_s=\sqrt[4]{\frac{S_0[2(1-R)-A_s]}{4\sigma(2-A_l)}}$

大气温度： $T_a=\sqrt[4]{\frac{S_0[A_s+A_l(1-R-A_s)]}{4\sigma A_l(2-A_l)}}$

“大气+地表”辐射平衡

- 取 $A_l = 0.9$，$A_s = 0.2$，$R = 0.3$，则 $T_s = 284.8\ K$，$T_a = 250.9\ K$。

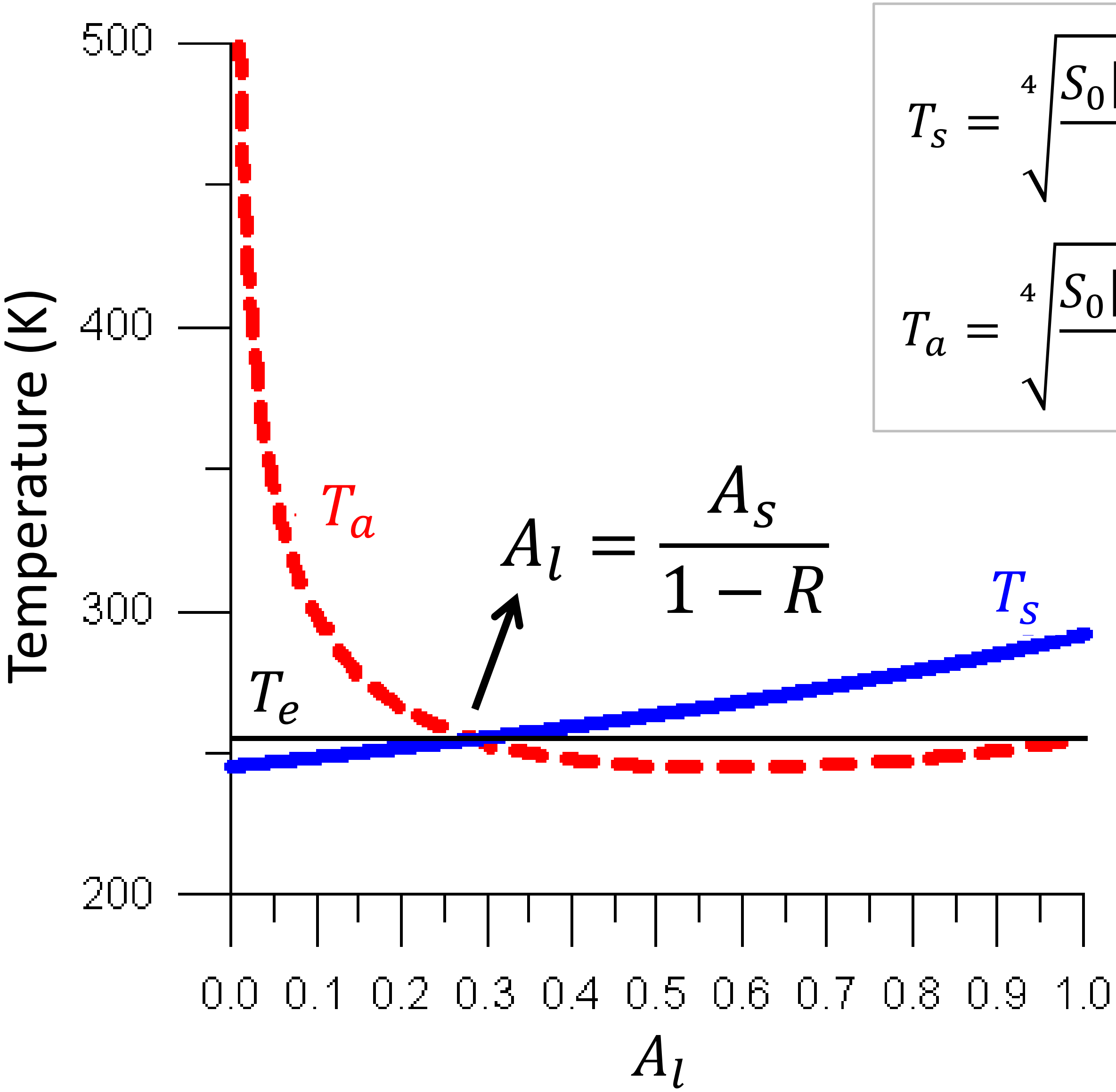
  可见，大气层的存在使地面平衡温度高于全球的等效温度 T_e，同时大气平均温度却低于 T_e

- 当 A_l 增大时，地面平衡温度也将升高。

  大气中温室气体的含量增加将使 A_l 增大，从而导致地面增温（温室效应增强）

- 当 A_l 增大时，大气平衡温度变化复杂，为什么？

“大气+地表”辐射平衡



$$T_s = \sqrt[4]{\frac{S_0[2(1-R)-A_s]}{4\sigma(2-A_l)}}$$

$$T_a = \sqrt[4]{\frac{S_0[A_s + A_l(1-R-A_s)]}{4\sigma A_l(2-A_l)}}$$

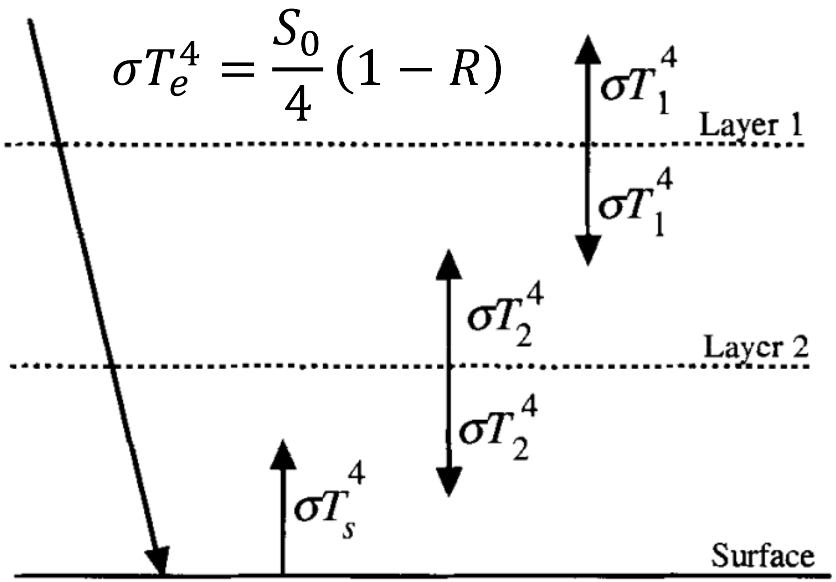
Multi-layer Atmosphere in Radiative Equilibrium

If the atmosphere is divided into multiple layers

- Each layer is opaque (black body) for long wave radiation
- Each layer is transparent for solar radiation (ignore O_3)

We can get surface and air temperatures as follows, assuming the Earth is in radiative equilibrium.

2-layer model



$$\sigma T_e^4 = \frac{S_0}{4}(1 - R)$$

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Multi-layer Atmosphere in Radiative Equilibrium

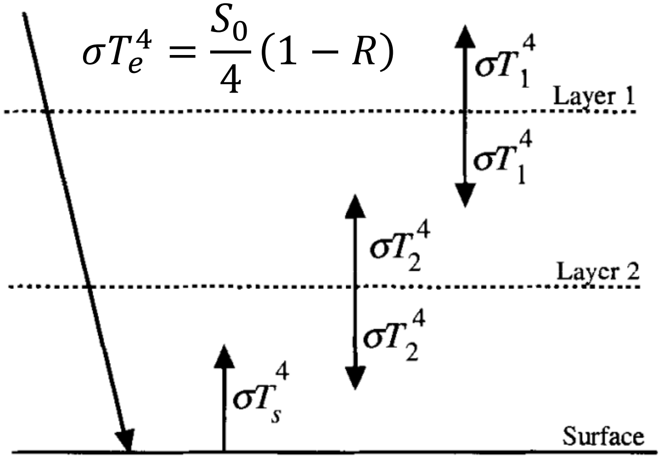
TOA: $$\frac{S_0}{4}(1-R) = \sigma T_e^4 = \sigma T_1^4$$

Layer 1: $$\sigma T_2^4 = 2\sigma T_1^4$$

Layer 2: $$\sigma T_1^4 + \sigma T_s^4 = 2\sigma T_2^4$$

Surface: $$\frac{S_0}{4}(1-R) + \sigma T_2^4 = \sigma T_s^4$$

2-layer model



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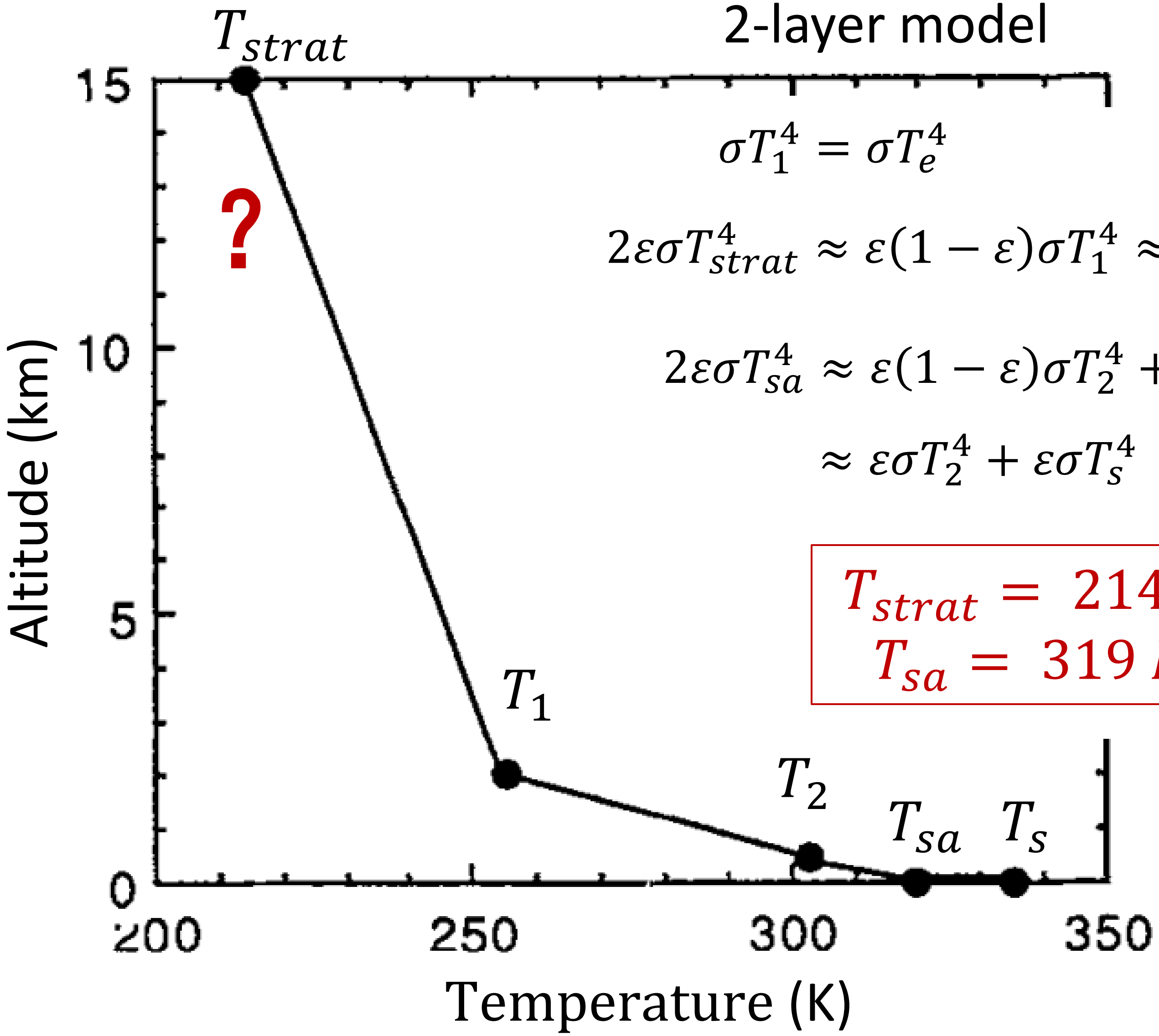
We get: $$T_s = \sqrt[4]{\frac{3S_0(1-R)}{4\sigma}} = \sqrt[4]{3}T_e \qquad T_2 = \sqrt[4]{2}T_e$$

For a n-layer model: $$T_s = \sqrt[4]{n+1}T_e \qquad T_n = \sqrt[4]{n}T_e$$

Multi-layer Atmosphere in Radiative Equilibrium

2-layer model



$$\sigma T_1^4 = \sigma T_e^4$$

$$2\varepsilon\sigma T_{strat}^4 \approx \varepsilon(1-\varepsilon)\sigma T_1^4 \approx \varepsilon\sigma T_1^4$$

$$2\varepsilon\sigma T_{sa}^4 \approx \varepsilon(1-\varepsilon)\sigma T_2^4 + \varepsilon\sigma T_s^4$$

$$\approx \varepsilon\sigma T_2^4 + \varepsilon\sigma T_s^4$$

$$\boxed{\begin{array}{c} T_{strat} = 214\ K \\ T_{sa} = 319\ K \end{array}}$$

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Radiative Heating for Each Layer

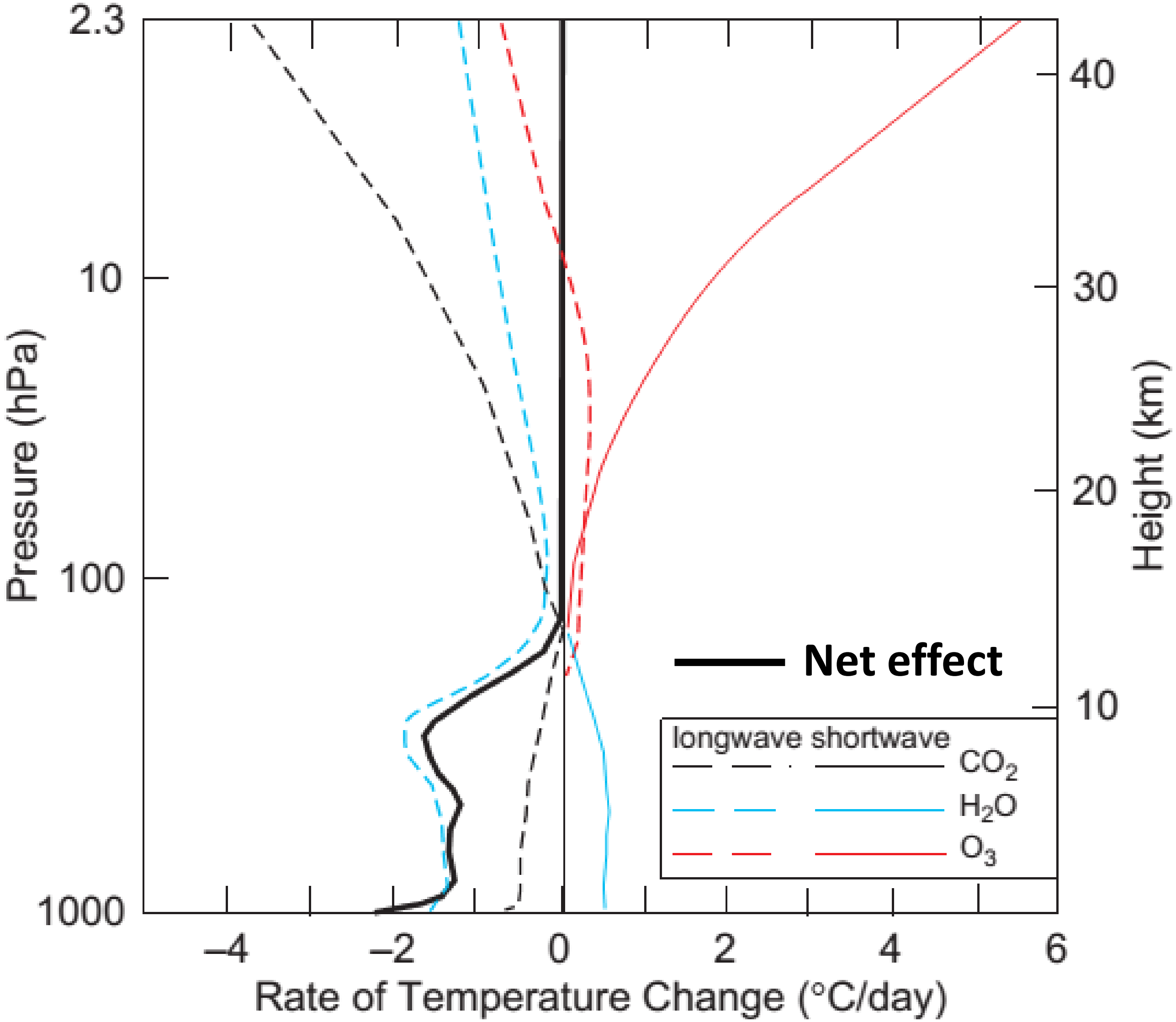
- For a thin layer of atmosphere (from z to $z + dz$, with the same temperature), the radiative flux density at its lower and upper edges are $F(z)$ and $F(z + dz)$, respectively.
- Thus, the rate of energy change for the layer:

$$\rho \frac{\delta q}{\delta t} dz \cdot \Delta S = \rho c_p \frac{\partial T}{\partial t} dz \cdot \Delta S = -\frac{\partial F}{\partial z} dz \cdot \Delta S$$

And the rate of temperature change due to radiative energy imbalance (°C$/day$):

$$\frac{\partial T}{\partial t} = -\frac{86400}{\rho c_p} \frac{\partial F}{\partial z}$$

Radiative Heating Rate



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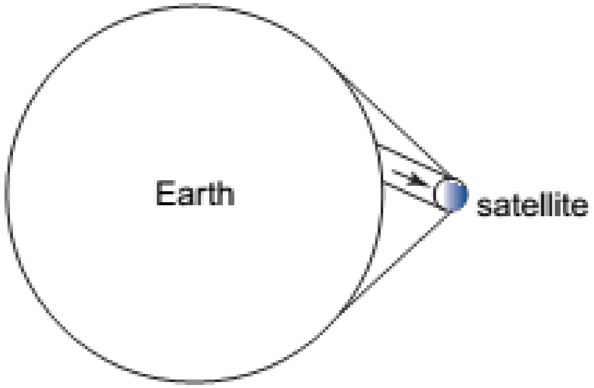
思考题



A. A small, perfectly black, spherical satellite is in orbit around the Earth at an altitude of 2000 km. What angle does the Earth subtend when viewed from the satellite?

B. If the Earth radiates as a blackbody at an equivalent blackbody temperature $T_e = 255\ K$, calculate the radiative equilibrium temperature of the satellite when it is in the Earth's shadow.

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思考题

A. Consider two opaque walls facing each other. One of the walls is a blackbody and the other wall is “gray” (i.e.,α_λ independent of λ). The walls are initially at the same temperature T and, apart from the exchange of radiation between them, they are thermally insulated from their surroundings. If α and ε are the absorptivity and emissivity of the gray wall, prove that $\varepsilon = \alpha$.

B. Consider the situation where two gray walls are facing each other. One wall has absorptivity α_1 and the other α_2. Prove that

$$\frac{F_1'}{\alpha_1} = \frac{F_2'}{\alpha_2}$$

where F_1' and F_2' are the flux densities of the radiation emitted from the two plates. Make sure of the fact that the two plates are in radiative equilibrium at the same temperature.

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温室效应、温室气体

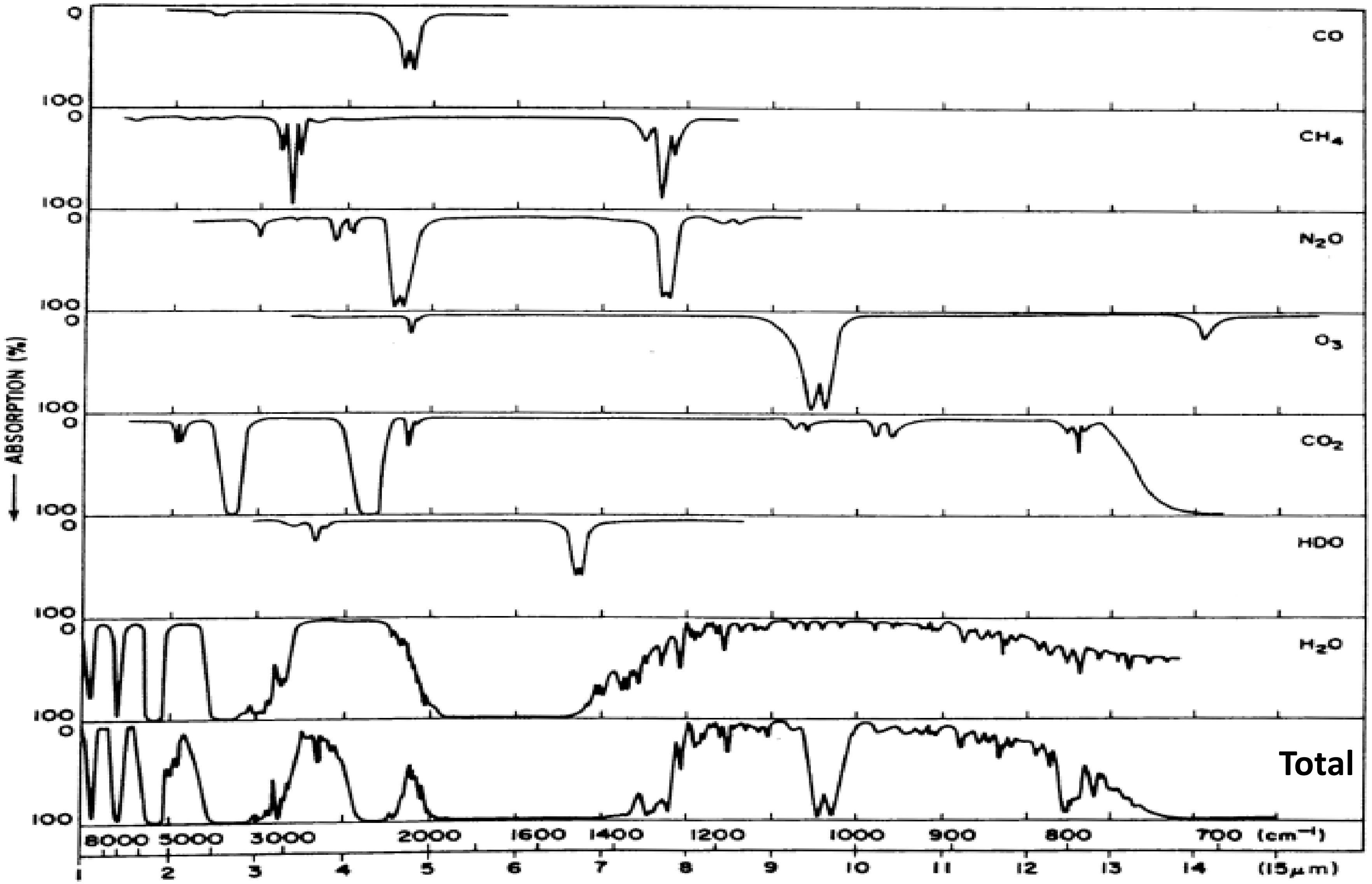
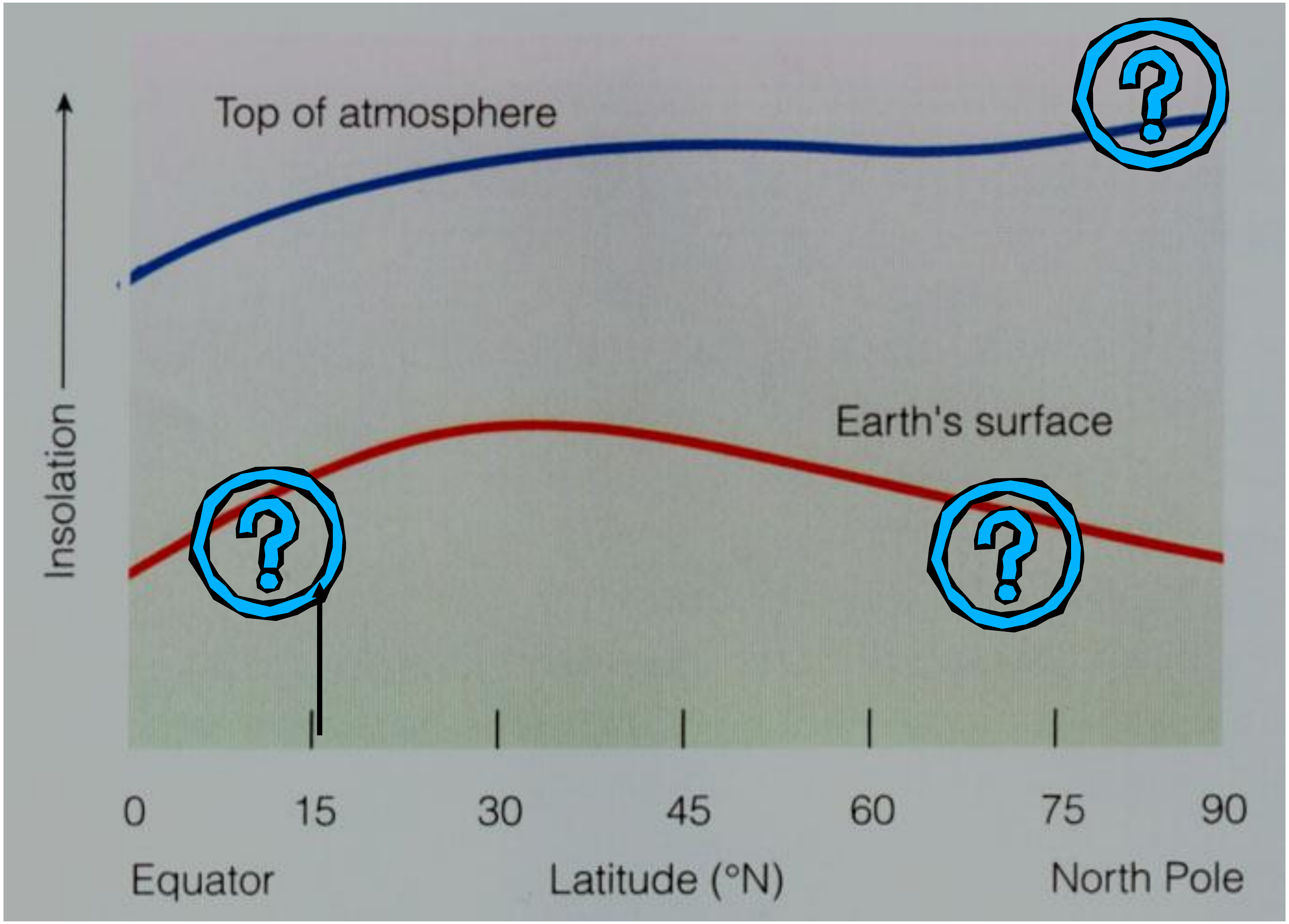


Figure 7.3 Low-resolution IR absorption spectra of the major atmospheric gases.

Incident Solar Radiation on Summer Solstice



地球系统辐射平衡

地气系统的有效温度取决于二个因子

- 太阳常数：主要由太阳温度和日地平均距离决定。
- 地气系统的行星反照率：它与地气系统的许多特性，如海洋的反射、陆面的反射、大气散射和云反射有关。

在太阳系中，几颗行星距太阳的距离不同，它们收到的太阳辐射的情况也不同。

- 下表给出地球及其相邻的五颗行星离太阳的距离和它们相应的太阳常数。
- 假设它们的行星反照率均为 $A = 0.3$，它们的有效温度都各自是多少?在表中也给出目前已测到的各颗行星的行星反照率。请进一步计算它们各自的有效温度。